NEWTON'S LAWS OF MOTION

1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :

(1) may change only speed
(2) may change only direction of motion.
(3) may change both the speed and direction of motion.
(4) may change size and shape of a body

Unit of force : Newton and $\frac{kg \cdot m}{s^2}$ (MKS System)

dyne and $\frac{g \cdot cm}{s^2}$ (CGS System)

1 Newton = 10^5 dyne

Kilogram force (kgf) : The force with which earth attracts a 1kg body towards its centre is called kilogram force, thus

$$\text{kgf} = \frac{\text{Forece in newton}}{g}$$

Dimensional Formula of force : $[MLT^{-2}]$

1.1 Fundamental Forces

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

(A) Gravitational Force : The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.

$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r} \qquad S \xrightarrow{\vec{r}} T$$

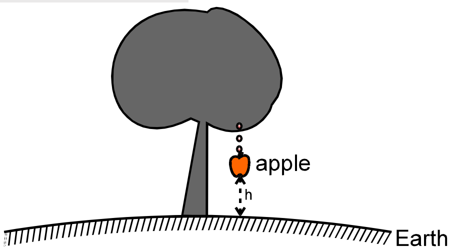
= position vector of test particle 'T' with respect to source particle 'S'. and G = universal gravitational constant = 6.67×10^{-11} Nm^2/kg^2.

(i) It is the weakest force and is always attractive.

(ii) It is a long range force as it acts between any two particles situated at any distance in the universe.

(iii) It is independent of the nature of medium between the particles.

An apple is freely falling as shown in figure, When it is at a height h, force between earth and apple is given by



$$F = \frac{GM_e m}{(R_e + h)^2}$$

where M_e – mass of earth, R_e – radius of earth. It acts towards earth's centre. Now rearranging above result,

$$F = m\frac{GM_e}{R_e^2} \cdot \left(\frac{R_e}{R_e + h}\right)^2.$$

$$F = mg\left(\frac{R_e}{R_e + h}\right)^2 \left\{g = \frac{GM_e}{R_e^{\,2}}\right\}$$

Here h << R_e, so $\frac{R_e}{R_e + h} \approxeq 1$ $\quad\therefore\quad F = mg$

This is the force exerted by earth on any particle of mass m near the earth surface. The value of $g = 9.81\ m/s^2 \simeq 10\ m/s^2\ \pi^2\ m/s^2 \simeq 32\ ft/s^2$. It is also called acceleration due to gravity near the surface of earth.

(B) Electromagnetic Force : Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

(a) These can be attractive or repulsive.

(b) These are long range forces

(c) These depend on the nature of medium between the charged particles.

(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

(C) Nuclear Force : It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

(D) Weak Force : It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

It has been found that for two protons at a distance of 1 Fermi :

$F_N : F_{EM} : F_W : F_G : : 1 : 10^{-2} : 10^{-7} : 10^{-38}$

1.2 Classification of forces on the basis of contact :

(A) Field Force : Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force. Examples

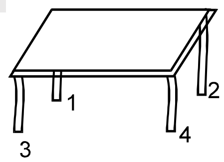
(a) Gravitation force (b) Electromagnetic force

(B) Contact Force : Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

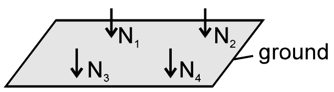
Examples :

(a) Normal force (N) :

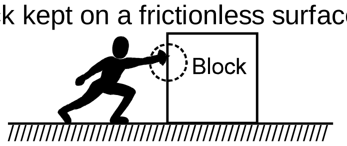
It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force. A table is placed on Earth as shown in figure



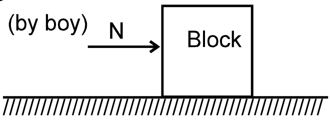
Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



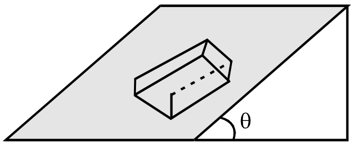
Now a boy pushes a block kept on a frictionless surface.



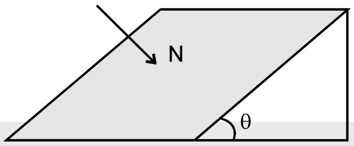
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



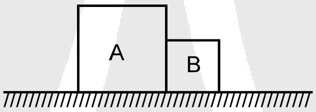
Normal force exerted by block on the surface of inclined plane is shown in figure.



Force acts perpendicular to the surface

Solved Examples

Example 1. Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.

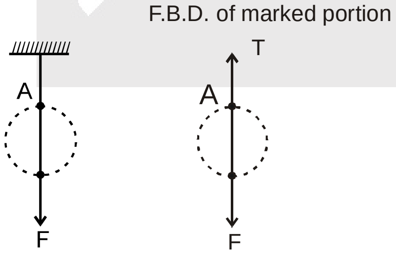


Solution : In above problem, block A does not push block B, so there is no molecular interaction between A and B. Hence normal force exerted by A on B is zero.

Note : Normal is a dependent force, it comes in role when one surface presses the other.

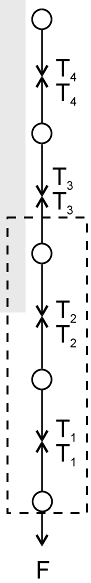
(b) Tension :

Tension in a string is a electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.

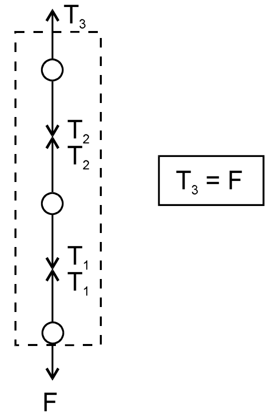


$\Rightarrow$ T = F

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in figure. The attraction force between two segments is equal and opposite due to Newton's third law.



For calculating tension at any segment, we consider two or more than two parts as a system.



Here interaction between segments are considered as internal forces, so they are not shown in F.B.D.

(C) Frictional force : It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.

2. THIRD LAW OF MOTION :

To every action, there is always an equal and opposite reaction. Newton's law from an 1803 translation from Latin as Newton wrote

"To every action there is always opposed an equal and opposite reaction : to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

2.1 Important points about the Third Law

(a) The terms 'action' and 'reaction' in the Third Law mean nothing else but 'force'. A simple and clear way of stating the Third Law is as follows : Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

(b) The terms 'action' and 'reaction' in the Third Law may give a wrong impression that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause-effect relation implied in the Third Law. The force on A by B and the force on B by A act at the same instant. Any one of them may be called action and the other reaction.

(c) Action and reaction forces act on different bodies, not on the same body. Thus if we are considering the motion of any one body (A or B), only one of the two forces is relevant. It is an error to add up the two forces and claim that the net force is zero.
However, if you are considering the system of two bodies as a whole, F_{AB} (force on A due to B) and F_{BA} (force on B due to A) are internal forces of the system (A + B). They add up to give a null force. Internal forces in a body or a system of particles thus cancel away in pairs. This is an important fact that enables the Second Law to be applicable to a body or a system of particles.

3. SYSTEM :

Two or more than two objects which interact with each other form a system.

3.1 Classification of forces on the basis of boundary of system :

(A) Internal Forces : Forces acting each with in a system among its constituents.

(B) External Forces : Forces exerted on the constituents of a system by the outside surroundings are called as external forces.

(C) Real Force : Force which acts on an object due to other object is called as real force. An isolated object (far away from all objects) does not experience any real force.

4. FREE BODY DIAGRAM

A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

4.1 Steps for F.B.D.

Step 1 : Identify the object or system and isolate it from other objects clearly specifying its boundary.

Step 2 : First draw non-contact external force in the diagram. Generally it is weight.

Step 3 : Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.

In F.B.D, internal forces are not drawn, only external are drawn.

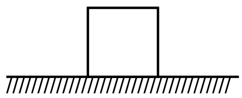
Solved Examples

Example 2. A block of mass 'm' is kept on the ground as shown in figure.

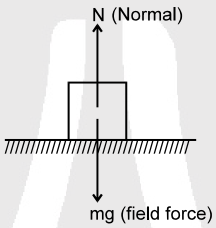
(i) Draw F.B.D. of block.

(ii) Are forces acting on block action–reaction pair.

(iii) If answer is no, draw action reaction pair.

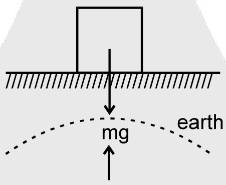


Solution : (i) F.B.D. of block

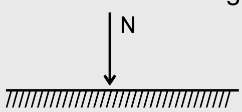


(ii) 'N' and mg are not action-reaction pair. Since pair act on different bodies, and they are of same nature.

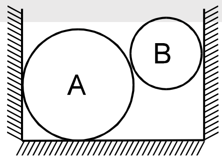
(iii) Pair of 'mg' of block acts on earth in opposite direction.



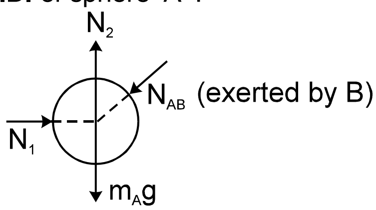
and pair of 'N' acts on surface as shown in figure.

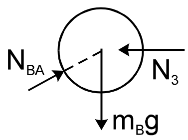


Example 3. Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



Solution : **F.B.D.** of sphere 'A' :





F.B.D. of sphere 'B' : (exerted by A)

Note : Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

5. NEWTON'S LAWS OF MOTION :

5.1 First Law of Motion

Each body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Newton's first law is really a statement about reference frames in that it defines the types of reference frames in which the laws of Newtonian mechanics hold. From this point of view the first law is expressed as:

If the net force acting on a body is zero, it is possible to find a set of reference frames in which that body has no acceleration.

Newton's first law is sometimes called the law of inertia and the reference frames that it defines are called inertial reference frames.

Newton's law from an 1803 translation from Latin as Newton wrote

"Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."

Examples of this law :

(a) A bullet fired on a glass window makes a clean hole through it while a stone breaks the whole of it. The speed of bullet is very high. Due to its large inertia of motion, it cuts a clean hole through the glass. When a stone is thrown, it inertia is much lower so it cannot cut through the glass.

(b) A passenger sitting in a bus gets a jerk when the bus starts or stops suddenly.

5.2 Second Law of Motion :

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's law from an 1803 translation from Latin as Newton wrote

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

Mathematically

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Or $\vec{F} = m\vec{a}$

where $\vec{p} = m\vec{v}$, $\vec{p}$ = Linear momentum.

In case of two particles having linear momentum $\vec{P}_1$ and $\vec{P}_2$ and moving towards each other under mutual forces, from Newton's second law ;

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = \vec{F} = 0 \quad \Rightarrow \quad \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$

$\vec{F}_1 + \vec{F}_2 = 0 \quad \Rightarrow \quad \vec{F}_2 = -\vec{F}_1$ which is Newton's third law.

5.3 Important points about second law

(a) The Second Law is obviously consistent with the First Law as F = 0 Implies a = 0.

(b) The Second Law of motion is a vector law. It is actually a combination of three equations, one for each component of the vectors :

$$F_x = \frac{dp_x}{dt} = ma_x \qquad F_y = \frac{dp_y}{dt} = ma_y \qquad F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

(c) The Second Law of motion given above is strictly applicable to a single point mass. The force **F** in the law stand for the net external force on the particle and a stands for the acceleration of the particle. Any internal forces in the system are not to be included in F.

(d) The Second Law of motion is a local relation. What this means is that the force F at a point in space (location of the particle) at a certain instant of time is related to a at the same point at the same instant. That is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

5.4 Applications of Newton's Laws

(a) When objects are in equilibrium

To solve problems involving objects in equilibrium:

Step 1 : Make a sketch of the problem.

Step 2 : Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.

Step 3 : Choose a convenient coordinate system and resolve all forces into rectangular components along x and y direction.

Step 4 : Apply the equations $\sum F_x = 0$ and $\sum F_y = 0$.

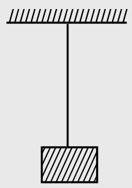
Step 5 : Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6 : If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Solved Examples

Example 4. A 'block' of mass 10 kg is suspended with string as shown in figure. Find tension in the string. ($g = 10\ m/s^2$)

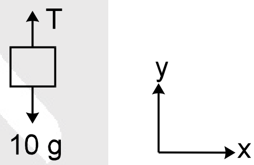


Solution : F.B.D. of block

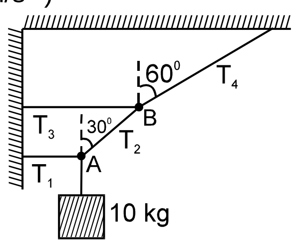
$\Sigma F_y = 0$

$T - 10\ g = 0$

$\therefore \quad T = 100\ N$



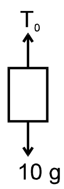
Example 5. The system shown in figure is in equilibrium. Find the magnitude of tension in each string ; T_1, T_2, T_3 and T_4. ($g = 10\ m/s^{-2}$)



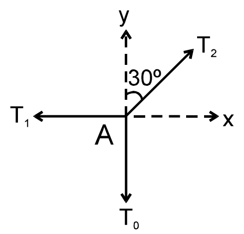
Solution : **F.B.D. of block 10 kg**

$T_0 = 10\ g$

$T_0 = 100\ N$



F.B.D. of point 'A'



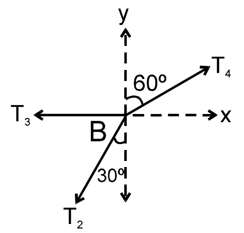
$\Sigma F_y = O$

$T_2 \cos 30^o = T_0 = 100$ N $\quad T_2 = \dfrac{200}{\sqrt{3}}$ N

$\Sigma F_x = O$

$T_1 = T_2 \sin 30^o = \dfrac{200}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{100}{\sqrt{3}}$ N.

F.B.D. of point 'B'

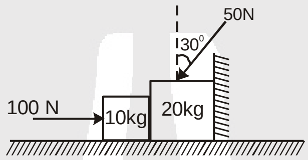


$\Sigma F_y = O \quad \Rightarrow \quad T_4 \cos 60^0 = T_2 \cos 30^0$

and $\Sigma F_x = O \quad \Rightarrow T_3 + T_2 \sin 30^0 = T_4 \sin 60^0$

$T_3 = \dfrac{200}{\sqrt{3}}$ N, $\quad T_4 = 200$ N

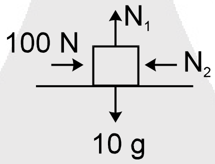
Example 6. Two blocks are kept in contact as shown in figure. Find



(a) forces exerted by surfaces (floor and wall) on blocks.

(b) contact force between two blocks.

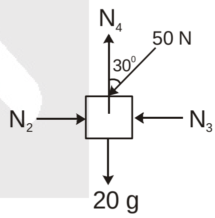
Solution : **F.B.D. of 10 kg block**



$N_1 = 10\text{ g} = 100$ N .....(1)

$N_2 = 100$ N .....(2)

F.B.D. of 20 kg block



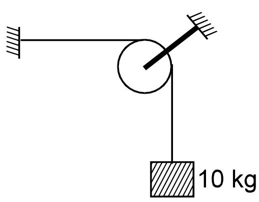
$N_2 = 50 \sin 30^o + N_3$

$\therefore \quad N_3 = 100 - 25 = 75$ N .....(3)

and $N_4 = 50 \cos 30° + 20$ g

$N_4 = 243.30$ N

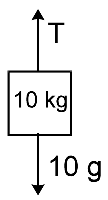
Example 7. Find magnitude of force exerted by string on pulley.

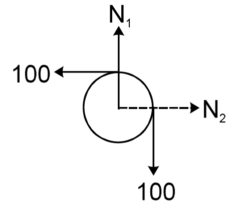


Solution : **F.B.D. of 10 kg block :** T = 10 g = 100 N

F.B.D. of pulley : Since string is massless, so tension in both sides of string is same. Force exerted by string

$= \sqrt{(100)^2 + (100)^2} = \sqrt{2}\,100$ N





Note : Since pulley is in equilibrium position, so net forces on it is zero.

Hence force exerted by hinge on it is $100\sqrt{2}$ N.

(b) Accelerating Objects

To solve problems involving objects that are in accelerated motion :

Step 1 : Make a sketch of the problem.

Step 2 : Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not to include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

Step 3 : Choose a convenient coordinate system, show location of coordinate axes explicitly in the free-body diagram, and then determine components of forces with reference to these axes and resolve all forces into x and y components.

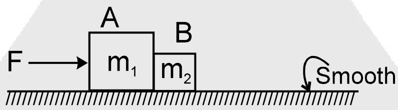
Step 4 : Apply the equations $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$

Step 5 : Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

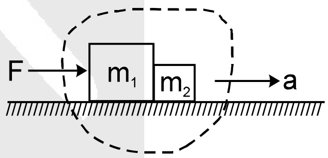
Step 6 : If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps. Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Solved Examples

Example 8. A force F is applied horizontally on mass m_1 as shown in figure. Find the contact force between m_1 and m_2.

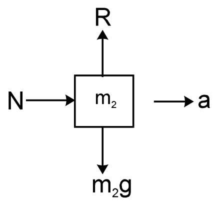


Solution : Considering both blocks as a system to find the common acceleration. Common acceleration



$$a = \frac{F}{(m_1 + m_2)} \quad .....(1)$$

To find the contact force between 'A' and 'B' we draw F.B.D. of mass m_2.



F.B.D. of mass m_2

$\Sigma F_x = ma_x$

$N = m_2 . a$

$$N = \frac{m_2 F}{(m_1 + m_2)}$$

Example 9. The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

Solution : From second law of motion :

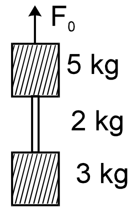
$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = 2.\frac{d}{dt}\,(at\hat{i} + bt^2\hat{j}) \Rightarrow \vec{F} = 2a\hat{i} + 4bt\hat{j}$$

Example 10. A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s^2 by an external force F_0.

(a) What is F_0 ?

(b) What is the net force on rope ?

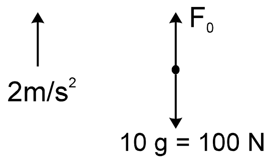
(c) What is the tension at middle point of the rope ? (g = 10 m/s^2)



Solution : For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system

(a)



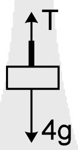
$F_0 - 100 = 10 \times 2$

F = 120 N .....(1)

(b) According to Newton's second law, net force on rope.

F = ma = (2) (2) = 4 N .....(2)

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.

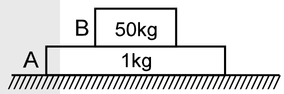


T – 4g = 4.(2) ; T = 48 N

Example 11. A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 1kg block. (All surface are smooth). Find (g = 10 m/s^2)

(a) Acceleration of block A and B.

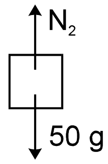
(b) Force exerted by B on A.



Solution : **(a) F.B.D. of 50 kg**

N_2 = 50 g = 500 N

along horizontal direction, there is no force $a_B = 0$



(b) F.B.D. of 1 kg block : along horizontal direction

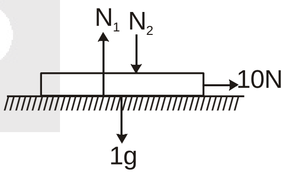
10 = 1 a_A.

a_A = 10 m/s^2

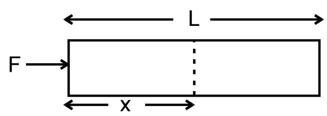
along vertical direction

$\therefore$ $N_1 = N_2 + 1g$

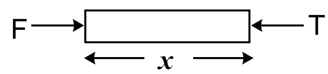
= 500 + 10 = 510 N



Example 12. A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance 'x' from the end where force is applied.



Solution : Considering rod as a system, we find acceleration of rod a = $\frac{F}{M}$

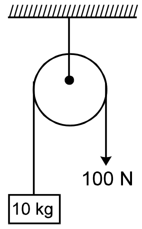


now draw F.B.D. of rod having length 'x' as shown in figure.

Using Newton's second law

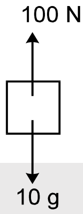
$$F - T = \left(\frac{M}{L}\right)x.a \Rightarrow T = F - \frac{M}{L}x.\frac{F}{M} \Rightarrow T = F\left(1 - \frac{x}{L}\right).$$

Example 13. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. ($g = 9.8\ m/s^2$)



Solution : Since string is pulled by 100 N force. So tension in the string is 100 N.

F.B.D. of 10 kg block

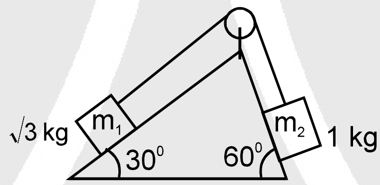


$100 - 10\,g = 10\,a$

$100 - 10 \times 9.8 = 10\,a$

$a = 0.2\ m/s^2$.

Example 14. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :

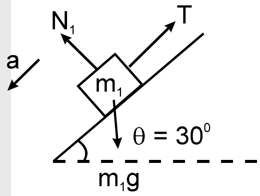


(i) Acceleration of mass m_1 and m_2

(ii) Tension in the string

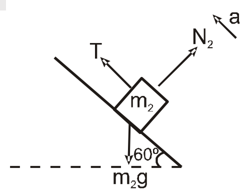
(iii) Net force on pulley exerted by string

Solution : **F.B.D. of m_1 :** $m_1 g\sin\theta - T = m_1 a$



$$\frac{\sqrt{3}}{2}\,g - T = \sqrt{3}\,a \qquad .....(1)$$

F.B.D. of m_2 : $T - m_2 g\sin\theta = m_2 a$



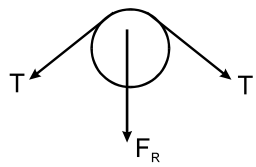
$$T - 1.\frac{\sqrt{3}}{2}\,g = 1.a \qquad .....(2)$$

Adding eq.(1) and (2) we get a = 0

Putting this value in eq.(i) we get

$$T = \frac{\sqrt{3}g}{2},$$

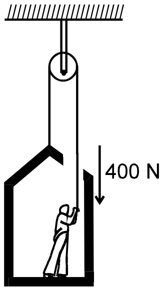
F.B.D. of pulley



$F_R = \sqrt{2}\,T$

$F_R = \sqrt{\frac{3}{2}}\,g$

Example 15. A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.

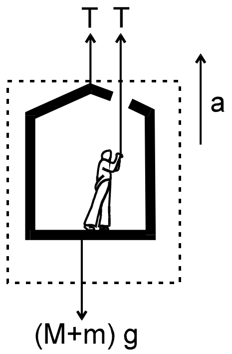


(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1s?

(iii) What force should he apply now to maintain the constant speed of 1 m/s?

Solution : The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulls the rope.



(i) Applying Newton's Second Law

$2T - (M + m)g = (M + m)a$

or $a = \dfrac{2T - (M+m)g}{M+m}$

Here M = 60 kg; m = 15 kg; T = 400 N

$g = 10 \text{ m/s}^2$

$a = \dfrac{2(400) - (60+15)(10)}{60+15} = 0.67 \text{ m/s}^2$

(ii) To attain a speed of 1 m/s in one second, the acceleration a must be 1 m/s^2.

Thus, the applied force is

$F = \dfrac{1}{2}(M + m)(g + a) = \dfrac{1}{2}(60 + 15)(10 + 1) = 412.5 \text{ N}$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

Thus, $2F - (M + m)g = 0$

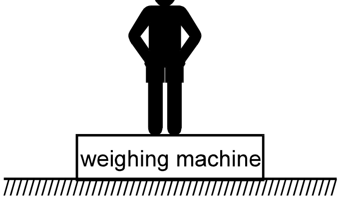
or $F = \dfrac{(M+m)g}{2} = \dfrac{(60+15)(10)}{2} = 375 \text{ N}$

6. WEIGHING MACHINE :

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

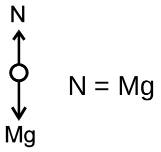
Solved Examples

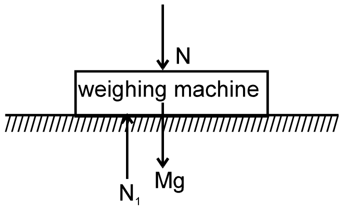
Example 16. A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine (g = 10 m/s^2).



Solution : For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man F.B.D. of weighing machine





Here force exerted by object on upper surface is N

Reading of weighing machine

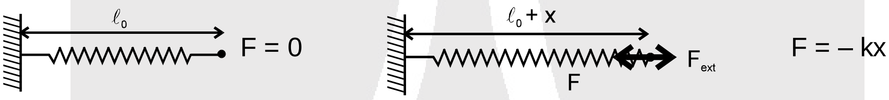
$N = Mg = 60 \times 10$

$N = 600$ N.

7. SPRING FORCE :

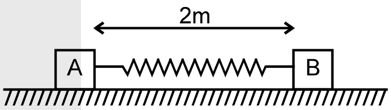
Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

When spring is in its natural length, spring force is zero.



Solved Examples

Example 17. Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m. Find spring force in following situations :



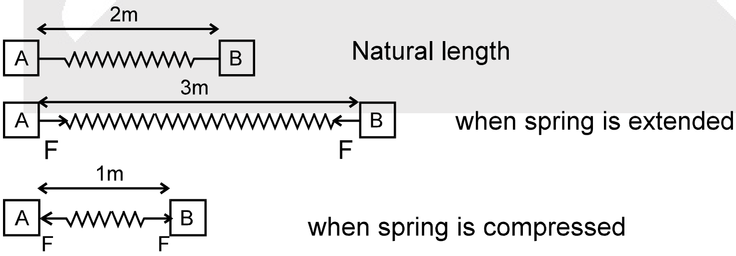
(a) If block 'A' and 'B' both are displaced by 0.5 m in same direction.

(b) If block 'A' and 'B' both are displaced by 0.5 m in opposite direction.

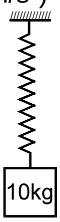
Solution : (a) Since both blocks are displaced by 0.5 m in same direction, so change in length of spring is zero. Hence, spring force is zero.

(b) In this case, change in length of spring is 1 m. In case of extension or compression of spring, spring force is $F = Kx = (200).(1)$

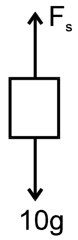
$F = 200$ N



Example 18. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10 \text{ m/s}^2$)



Solution : In this situation, spring is in extended state so spring force acts in upward direction. Let x be the extension in the spring.



F.B.D. of 10 kg block :

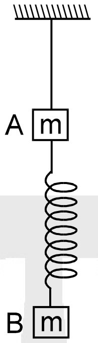
$F_s = 10g$

$\Rightarrow$ $Kx = 100$

$\Rightarrow$ $(100)x = (100)$

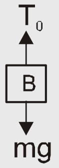
$\Rightarrow$ $x = 1m$

Example 19. Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.



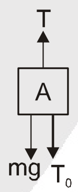
Solution : When block A and B are in equilibrium position

F.B.D of 'B'



$T_0 = mg$ .....(1)

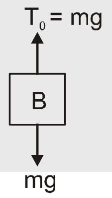
F.B.D of 'A'



$T = mg + T_0$
$T = 2\,mg$ .....(2)

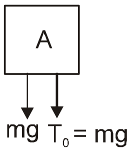
When string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of blocks A and B as shown in figure

F.B.D. of 'B'



$T_0 - mg = ma_B$
$a_B = 0$

F.B.D. of 'A'

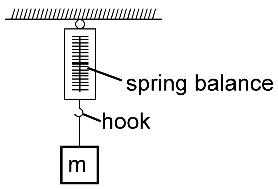


$mg + T_0 = ma_A$
$2mg = ma_A$
$a_A = 2g$ (downwards)

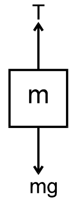
7.1 Spring Balance :

It does not measure the weight. It measures the force exerted by the object at the hook.

Symbolically, it is represented as shown in figure. A block of mass 'm' is suspended at hook.



When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.



F.B.D. of 'm'.

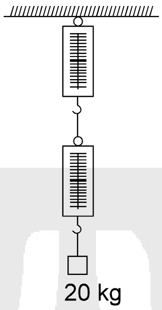
mg – T = 0

T = mg

Magnitude of T gives the reading of spring balance.

Solved Examples

Example 20. A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the

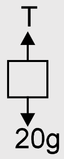


(1) reading of spring balance (1).

(2) reading of spring balance (2).

Solution : For calculating the reading, first we draw F.B.D. of 20 kg block.

F.B.D of 20 kg.



mg – T = 0

T = 20 g = 200 N

Since both balances are light so, both the scales will read 20 kg

8. CONSTRAINED MOTION:

8.1 String Constraint :

When two objects are connected through a string and if the string have the following properties:

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

Steps for String Constraint

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

(i) Object which moves along a line can be specified by one variable.

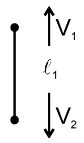
(ii) Object moving in a plane are specified by two variables.

(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format. $\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 + \ell_6 = \ell$

Step 4. Differentiate with respect to time

$$\frac{d\ell_1}{dt}+\frac{d\ell_2}{dt}+\frac{d\ell_3}{dt}+.... = 0$$

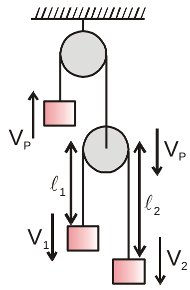


$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string $\frac{d\ell_1}{dt} = V_1 + V_2$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2.

Step 5. Repeat all above steps for different-different strings.

Let us consider a problem given below



Here $\ell_1 + \ell_2$ = constant

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$(V_1 - V_P) + (V_P - V_2) = 0$

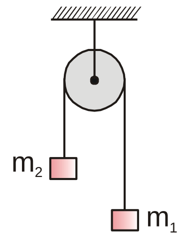
$V_p = \frac{V_1+V_2}{2}$ Similarly, $a_P = \frac{a_1+a_2}{2}$ Remember this result

Solved Examples

Example 21. Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find :

(i) the acceleration of each block

(ii) the tension in the string.

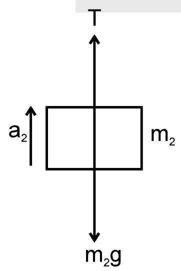


Solution : The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction.

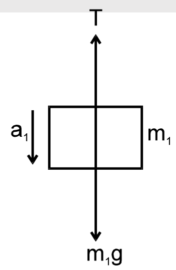
If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the figure.

F.B.D. of m_2 **F.B.D. of m_1**



F.B.D. of m_2



F.B.D. of m_1

Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $m_1g - T = m_1a_1$ ....(1)

Block m_2 $-m_2g + T = m_2a_2$ ....(2)

Number of unknowns : T, a_1 and a_2 (three)

Number of equations: only two

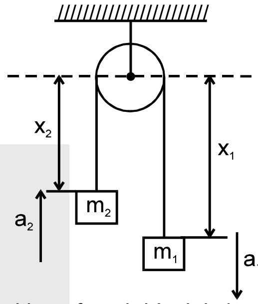
Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation ?

(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.



Position of each block is located w.r.t. centre of the pulley

Thus, $x_1 + x_2$ = constant

Differentiating both the sides w.r.t. time we get $\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations, therefore we differentiate it once again w.r.t. time.

Thus $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$\therefore \quad \frac{d^2x_1}{dt^2} = +a_1$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

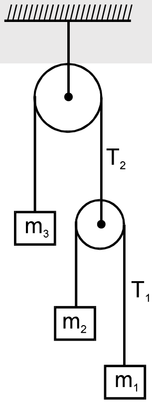
$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2$

Thus $a_1 - a_2 = 0$ or $a_1 = a_2 = a$ (say) is the required constraint relation.

Substituting $a_1 = a_2 = a$ in equations (1) and (2) and solving them, we get

(i) $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g$ (ii) $T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g$

Example 22. A system of three masses m_1, m_2 and m_3 are shown in the figure. The pulleys are smooth and massless; the strings are massless and inextensible.

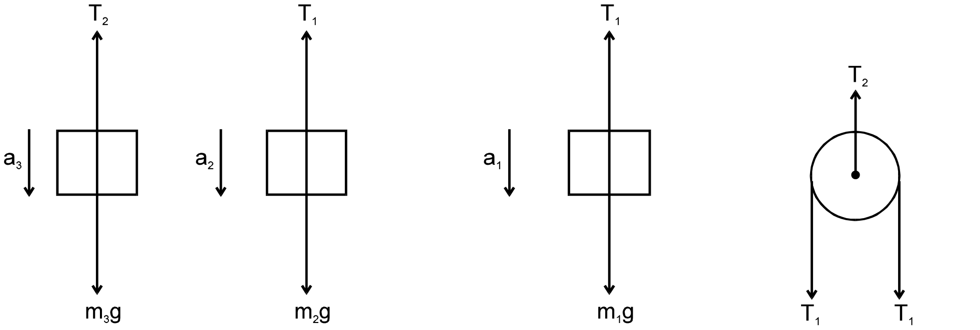


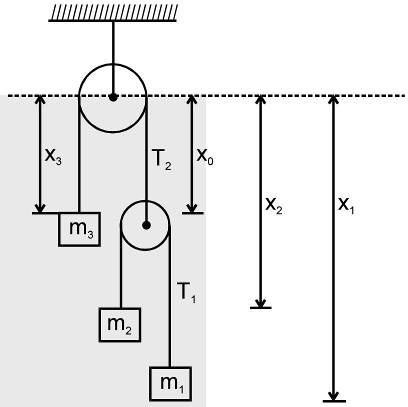
(i) Find the tensions in the strings.

(ii) Find the acceleration of each mass.

Solution : All the blocks are assumed to be moving downward and the free body diagram of each block is shown in figure.

F.B.D. m_3 **F.B.D. m_2** **F.B.D. m_1** **F.B.D. of pulley**





Applying Newton's Second Law to

Block m_1 : $m_1g - T_1 = m_1a_1$ ....(1)

Block m_2 : $m_2g - T_1 = m_2a_2$ ....(2)

Block m_3 : $m_3g - T_2 = m_3a_3$ ....(3)

Pulley : $T_2 = 2T_1$ ....(4)

Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations : Four

The constraint relation among accelerations can be obtained as follows

For upper string $x_3 + x_0 = c_1$

For lower string $x_2 - x_0) + (x_1 - x_0) = c_2$

$x_2 + x_1 - 2x_0 = c_2$

Eliminating x_0 from the above two relations,

we get $x_1 + x_1 + 2x_3 = 2c_1 + c_2$ = constant.

Differentiating twice with respect to time,

we get $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$

or $a_1 + a_2 + 2a_3 = 0$ ......(5)

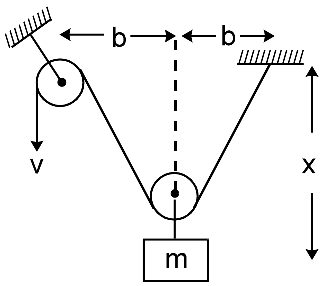
Solving equations (1) to (5), we get

(i) $T_1 = \left[\frac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$; $T_2 = 2T_1$

(ii) $a_1 = \left[\frac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$; $a_2 = \left[\frac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

$a_3 = \left[\frac{4m_1m_2 - m_3(m_1+m_2)}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

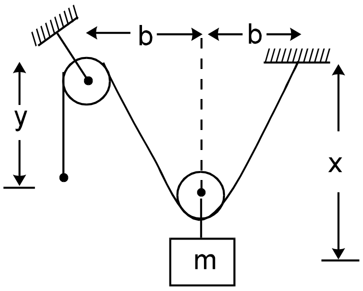
Example 23. The figure shows one end of a string being pulled down at constant velocity v. Find the velocity of mass 'm' as a function of 'x'.



Solution : Using constraint equation $2\sqrt{x^2 + b^2}$ + y = length of string = constant

Differentiating w.r.t. time : $\frac{2}{2\sqrt{x^2+b^2}} .2x \left(\frac{dx}{dt}\right) + \left(\frac{dy}{dt}\right) = 0$

$\Rightarrow \left(\frac{dy}{dt}\right) = v \Rightarrow \left(\frac{dx}{dt}\right) = -\frac{v}{2x}\sqrt{x^2+b^2}$



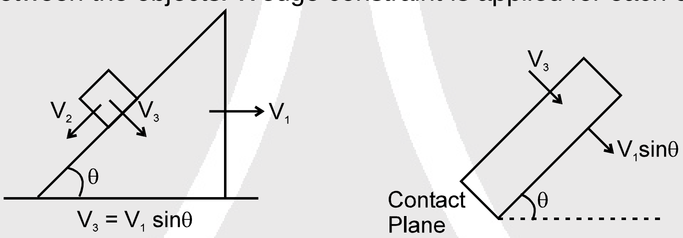
8.2 Wedge Constraint :

Conditions :

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.

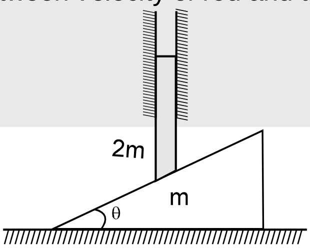


In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Solved Examples

Example 24. A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



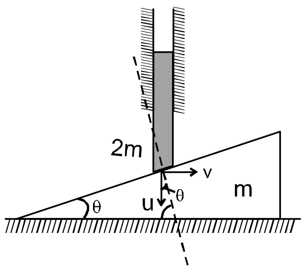
Solution : Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

$u \cos\theta = v \sin\theta$

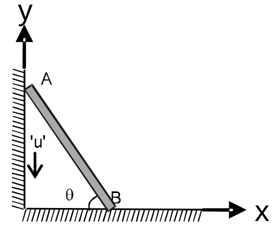
$\frac{u}{v} = \tan\theta$

$u = v \tan\theta$



perpendicular to contact of two blocks

Example 25. The velocity of end 'A' of rigid rod placed between two smooth vertical walls is 'u' along vertical direction. Find out the velocity of end 'B' of that rod, rod always remains in contact with the vertical wall and also find the velocity of centre of rod. also find equation of path of centre of rod.



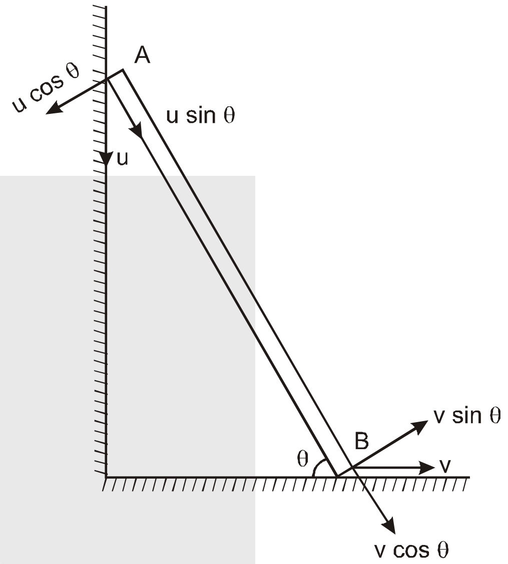
Solution : Since rod is rigid, its length can't increase.

∴ velocity of approach of A and B point of rod is zero.

⇒ $u \sin\theta - v\cos\theta = 0$

⇒ $v = u\tan\theta$

at any angle θ, x and y coordinates of center of mass are



Ans. $u\tan\theta.\ \left(\frac{u}{2}(-\hat{j}) + \frac{u\tan\theta}{2}\hat{i}\right)$, , $x^2 + y^2 = \left(\frac{\ell}{2}\right)^2$

9. NEWTON'S LAW FOR A SYSTEM

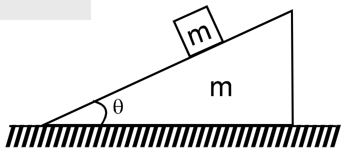
$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$

$\vec{F}_{ext} =$ Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

Solved Examples

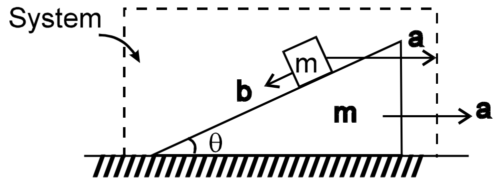
Example 26. The block of mass m slides on a wedge of mass 'm' which is free to move on the horizontal ground. Find the accelerations of wedge and block. (All surfaces are smooth).

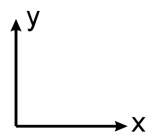


Solution : Let . a ⇒ acceleration of wedge

b ⇒ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction



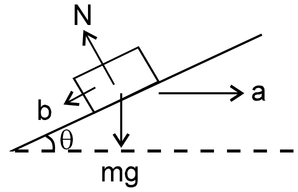


$F_x = m_1\,\vec{a}_{1x} + m_2\,\vec{a}_{2x} = 0$

$0 = ma + m(a - b \cos\theta)$ .....(1)

here 'a' and 'b' are two unknowns , so for making second equation, we draw F.B.D. of block.

F.B.D of block.



using Newton's second law along inclined plane

$mg \sin\theta = m (b - a \cos\theta)$ .....(2)

Now solving equations (1) and (2) we will get

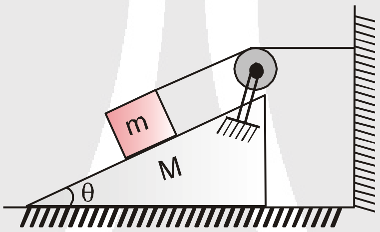
$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)} \text{ and } b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

So in vector form :

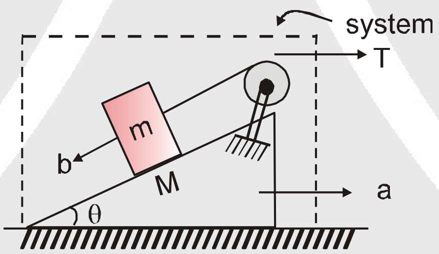
$$\vec{a}_{\text{wedge}} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i} \quad \Rightarrow \vec{a}_{\text{block}} = (a - b\cos\theta)\,\hat{i} - b\sin\theta\,\hat{j}$$

$$\vec{a}_{\text{block}} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\,\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\,\hat{j} \,.$$

Example 27. For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.

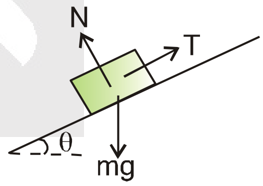


Solution : Considering block and wedge as a system and using. Newton's law for the system along x-direction



$T = Ma + m (a - b \cos\theta)$ .....(i)

F.B.D of m



along the inclined plane

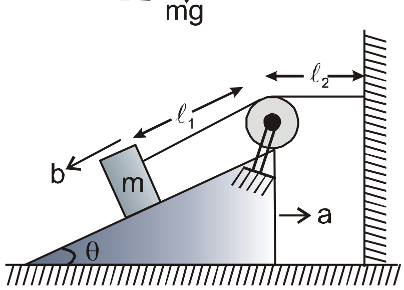
$mg \sin\theta - T = m (b - a \cos\theta)$ .....(ii)

using string constraint equation.

$\ell_1 + \ell_2 = \text{constant}$

$$\frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} = 0$$

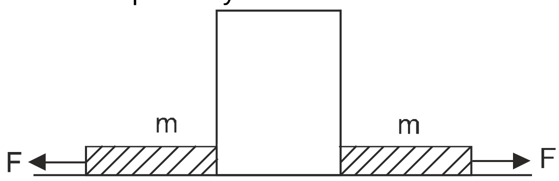
$b - a = 0$ .....(iii)



Solving above equations (i),(ii) & (iii) , we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

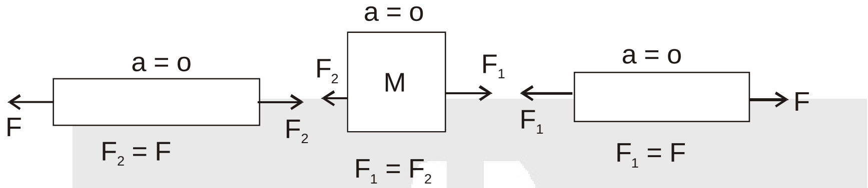
Example 28. A heavy block kept on a frictionless surface and being pulled by two ropes of equal mass m as shown in figure. At t = 0, the force on the left rope is withdrawn but the force on the right end continues to act. Let F_1 and F_2 be the magnitudes of the forces by the right rope and the left rope on the block respectively.



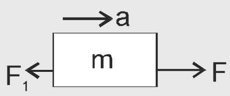
(A) $F_1 = F_2 = F$ for $t < 0$
(B) $F_1 = F_2 = F + mg$ for $t < 0$
(C) $F_1 = F$, $F_2 = F$ for $t > 0$
(D) $F_1 < F$, $F_2 = F$ for $t > 0$

Solution : For t < 0 As net force on system is zero, therefore acceleration of the system is zero

$\therefore$ $F_1 = F_2 = F$ for $t < 0$

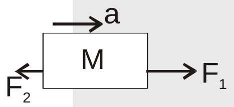


For t > 0 system is accelerated given by $a = \dfrac{F}{2m + M}$



$F - F_1 = ma$

$F_1 = F - ma$ .... (1)



$F_1 - F_2 = Ma$

$F_1 = F_2 + Ma$ .... (2)

$\boxed{F > F_1 > F_2}$

Ans. **(A)**

10. NEWTON'S LAW FOR NON INERTIAL FRAME :

$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non inertial frame

$\vec{F}_{Pseudo} = -m\,\vec{a}_{Frame}$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

10.1 Reference Frame:

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

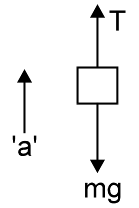
(a) **Inertial reference frame:** Frame of reference either stationary or moving with constant velocity.

(b) **Non-inertial reference frame:** A frame of reference moving with non-zero acceleration.

Solved Examples

Example 29. A lift having a simple pendulum attached with its ceiling is moving upward with constant acceleration 'a'. What will be the tension in the string of pendulum with respect to a boy inside the lift and a boy standing on earth, mass of bob of simple pendulum is m.

Solution : **F.B.D** . of bob (with respect to ground)



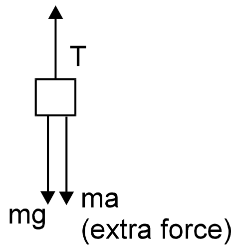
$$T - mg = ma$$

$T = mg + ma$ .....(i)

With respect to boy inside the lift, the acceleration of bob is zero. So he will write above equation in this manner.

$T - mg = m. (0)$ $\therefore$ $T = mg$

He will tell the value of tension in string is mg. But this is 'wrong'. To correct his result, he makes a free body diagram in this manner, and uses Newton's second law.



$T = mg + ma$ .....(ii)

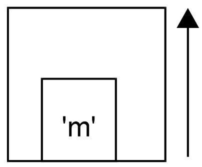
By using this **extra force**, equations (i) and (ii) give the same result. This **extra forc**e is called **pseudo force**. This **pseudo force** is used when a problem is solved with a accelerating frame (Non-inertial)

Note : Magnitude of Pseudo force = mass of system × acceleration of frame of reference.

Direction of force:

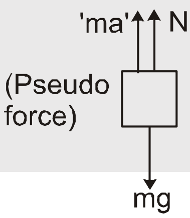
Opposite to the direction of acceleration of frame of reference, (not in the direction of motion of frame of reference)

Example 30. A box is moving upward with retardation 'a' <g, find the direction and magnitude of "pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



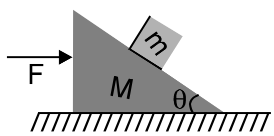
Solution : **Pseudo force** acts opposite to the direction of acceleration of reference frame.

pseudo force = ma in upward direction F.B.D of 'm' w.r.t. box (non–inertial)

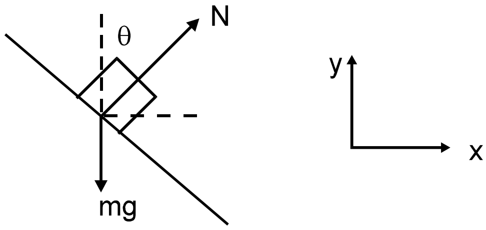


$$N + ma = mg$$
$$N = mg - ma$$

Example 31. All surfaces are smooth in the adjoining figure. Find F such that block remains stationary with respect to wedge.



Solution : Acceleration of (block + wedge) is a = $\frac{F}{(M+m)}$



Let us solve the problem by using both frames.

From inertial frame of reference (Ground)

F.B.D. of block w.r.t. ground (Apply real forces) : with respect to ground block is moving with an acceleration 'a' .

$\therefore \Sigma F_y = 0 \Rightarrow N \cos\theta = mg$ ....(i)

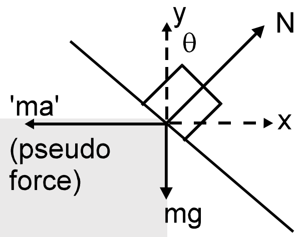
and $\Sigma F_x = ma \Rightarrow N \sin\theta = ma$ .....(ii)

From Eqs. (i) and (ii)

$a = g \tan\theta$

$\therefore F = (M + m) a = (M + m) g \tan\theta$

From non–inertial frame of reference (Wedge) :



F.B.D. of block w.r.t. wedge (real forces + pseudo force)

w.r.t. wedge, block is stationary

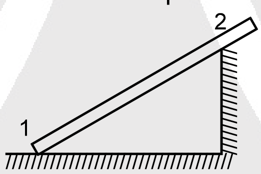
$\therefore \Sigma F_y = 0 \Rightarrow N \cos\theta = mg$ ....(iii)

$\Sigma F_x = 0 \Rightarrow N \sin\theta = ma$ ....(iv)

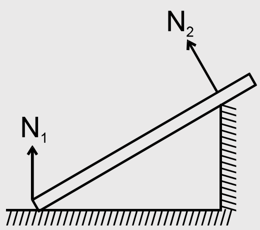
From Eqs. (iii) and (iv), we will get the same result i.e. $F = (M + m) g \tan\theta$

Solved Miscellaneous Problems

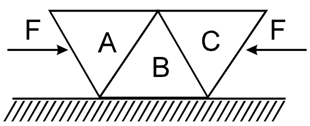
Problem 1. Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



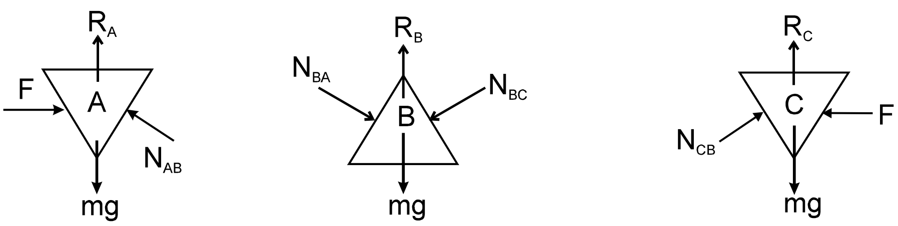
Solution : Normal force acts perpendicular to extended surface at point of contact..



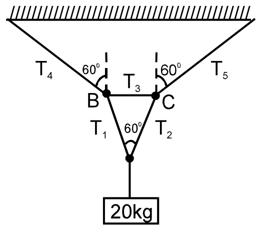
Problem 2. Three triangular blocks A, B and C of equal masses 'm' are arranged as shown in figure. Draw F.B.D. of blocks A,B and C. Indicate action–reaction pair between A,B and B,C.



Solution :

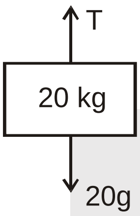


Problem 3. The system shown in figure is in equilibrium, find the tension in each string ; T_1, T_2, T_3, T_4 and T_5.



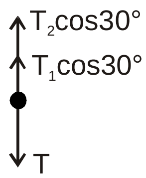
Answer : $T_1 = T_2 = \frac{200}{\sqrt{3}}$ N, $T_4 = T_5 = 200$ N, $T_3 = \frac{200}{\sqrt{3}}$ N.

Solution : **<u>FBD of 20 kg block</u>** →



So, $T = 20 \times g = 200$ N .....(1)

<u>From figure</u> →



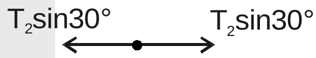
$T = T_1\cos30° + T_2\cos30°$ .....(2)

$T_1\sin30° = T_2\sin30°$

$T_1 = T_2$ .....(3)

so from equation (3) $T = 2T_1\cos30°$

$\mathbf{T_1 = \frac{200}{\sqrt{3}} = T_2}$ .....(4)



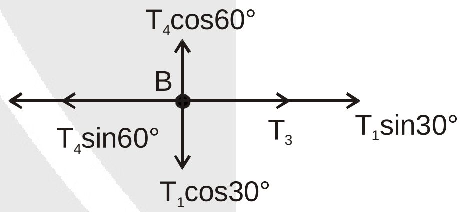
<u>From figure FBD of point B</u>

In vertical direction

So, $T_4\cos60° = T_1\cos30°$

$T_4 \times \frac{1}{2} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 200$ N

So, $\mathbf{T_4 = 200}$ **N**



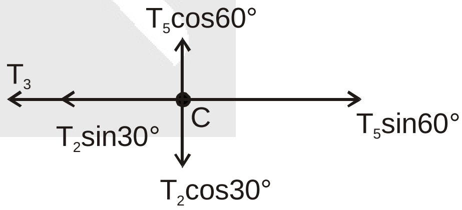
<u>FBD of point C</u>

Equating forces In vertical direction -

$T_5\cos60° = T_2\cos30°$

$T_5 \times \frac{1}{2} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$\mathbf{T_5 = 200}$ **N**



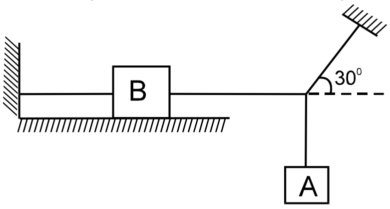
<u>For T_3</u> →

Equating forces in horizontal direction -

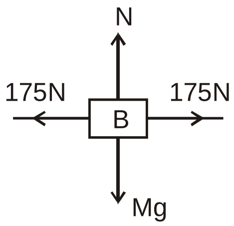
$T_3 + T_2\sin30° = T_5\sin60°$

$T_3 = 200 \times \frac{\sqrt{3}}{2} - \frac{200}{\sqrt{3}} \times \frac{1}{2} \Rightarrow \mathbf{T_3 = \frac{200}{\sqrt{3}}}$ **N**

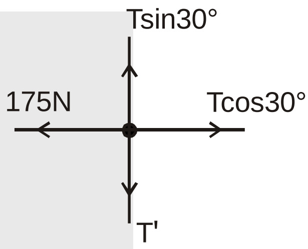
Problem 4. The breaking strength of the string connecting wall and block B is 175 N, find the magnitude of weight of block A for which the system will be stationary. The block B weighs 700 N. (g = 10 m/s^2)



Solution : **<u>FBD of block B</u>** $\rightarrow$



<u>FBD of point in figure</u> $\rightarrow$



Equating forces in horizontal direction $\rightarrow$

$T\cos 30° = 175$

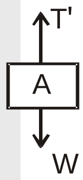
$T = \dfrac{175 \times 2}{\sqrt{3}}$ N

In vertical direction $\rightarrow$

$T\sin 30° = T'$

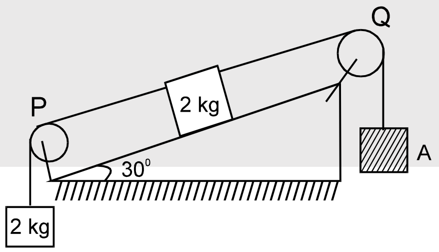
So, $T' = \dfrac{175 \times 2}{\sqrt{3}} \times \dfrac{1}{2} = \dfrac{175}{\sqrt{3}}$ N

<u>FBD of block A</u> $\rightarrow$



So, $\mathbf{T' = W = \dfrac{175}{\sqrt{3}}\ N}$

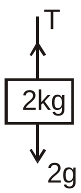
Problem 5. In the arrangement shown in figure, what should be the mass of block A so that the system remains at rest. Also find force exerted by string on the pulley Q. (g = 10 m/s^2)



Answer : m = 3 kg, $30\sqrt{3}$ N.

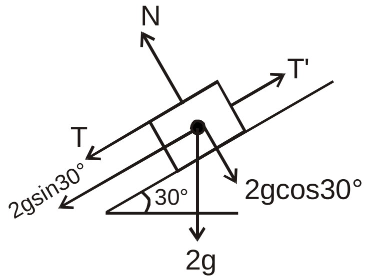
Solution : From figure

<u>FBD of 2 kg block hanging vertically</u> $\rightarrow$



T = 20 N .... (1)

FBD of 2kg block on incline plane

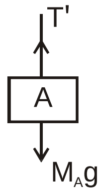


Along the plane →

$T + 2g\sin 30° = T'$

$T' = 20 + 20 \times \frac{1}{2} = 30$ N

FBD of block A

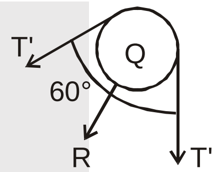


So $T' = M_Ag$

$M_A = \frac{T'}{g} = \frac{30}{10} = 3$ kg

$M_A = 3$ kg

FBD of pulley Q



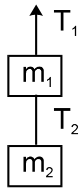
So, $R = 2T'\cos\frac{\theta}{2}$

$R = 2 \times 30 \cos 30°$

$R = 2 \times 30 \times \frac{\sqrt{3}}{2}$

$R = 30\sqrt{3}$ N

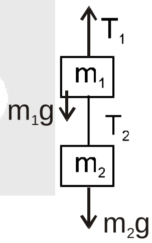
Problem 6. Two blocks with masses $m_1 = 0.2$ kg and $m_2 = 0.3$ kg hang one under other as shown in figure. Find the tensions in the strings (massless) in the following situations (g = 10 m/s^2)



(a) the blocks are at rest
(b) they move upward at 5 m/s
(c) they accelerate upward at 2 m/s^2
(d) they accelerate downward at 2 m/s^2
(e) if maximum allowable tension is 10 N. What is maximum possible upward acceleration ?

Answer : (a) 5 N, 3 N (b) 5 N, 3 N (c) 6 N, 3.6 N (d) 4 N, 2.4 N (e) 10 m/s^2

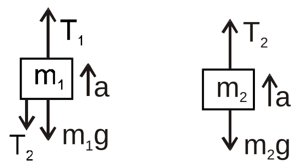
Solution : (a) At rest $a = 0$



$T_2 = m_2g = 0.3 \times 10 = 3$N
$T_1 = m_1g + T_2$
$T_1 = 0.2 \times 10 + 3 = 5$ N

(b) same as above
$a = 0$, $T_2 = 3$N, $T_1 = 5$N

(c) $a = 2$m/s^2 ↑ (upward)



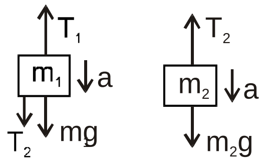
$T_2 - m_2g = m_2a$
$\Rightarrow$ $T_2 - m_2g = m_2a$
$\Rightarrow$ $T_2 - 0.3 \times 10 = 0.3 \times 2$
$\Rightarrow$ $T_2 = 0.6 + 3 = 3.6$ N
$T_1 - m_1g - T_2 = m_1a$
$\Rightarrow$ $T_1 - 0.2 \times 10 - 3.6 = 0.2 \times 2$
$\Rightarrow$ $T_1 = 0.4 + 5.6 = 6$N

(d) $a = 2$ m/s^2 (downward)
$m_2g - T_2 = m_2a$
$\Rightarrow$ $0.3 \times 10 - T_2 = 0.3 \times 2$
$\Rightarrow$ $T_2 = 3 - 0.6 = 2.4$ N

$T_2 + m_1g - T_1 = m_1a$

$\Rightarrow$ $2.4 + 2 - T_1 = 0.2 \times 2$

$\Rightarrow$ $T_1 = 4.4 - 0.4 =$ **4 N Ans.**



(e) Chance of breaking is of upper string means

$T_1 < 10$ N

For m_1 –

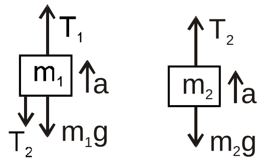
$T_1 - m_1g - T_2 = m_1a$

$10 - 2 - T_2 = 0.2$ a .....(1)

For m_2 –

$T_2 - m_2g = m_2a$

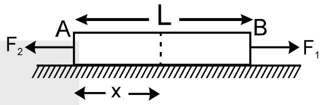
$\Rightarrow$ $T_2 - 3 = 0.3$ a .....(2)



Adding equation (1) and (2)

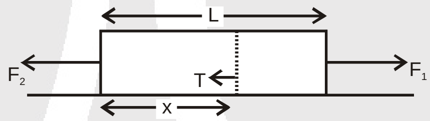
$8 - 3 = 0.5$ a $\Rightarrow$ $a = \frac{5}{0.5}$ = **10 m/s²**

Problem 7. Two forces F_1 and F_2 ($F_2 > F_1$) are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end 'A',

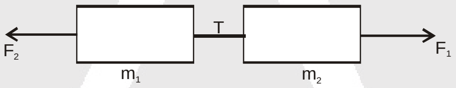


Answer : $T = F_2 - \frac{(F_2 - F_1)}{L}.x$

Solution : $a = \frac{F_2 - F_1}{m}$



$T - F_1 = m_2a$



$\Rightarrow$ $T - F_1 = \frac{m}{L}(L - x)\frac{F_2 - F_1}{m}$ $\quad (m_2 = \frac{m}{L}(L - x))$

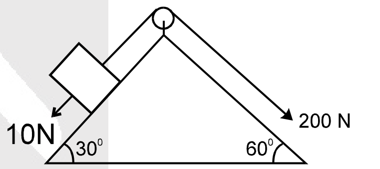
$\Rightarrow$ $T = F_1 + \left(1 - \frac{x}{L}\right)(F_2 - F_1) = F_1 + F_2 - F_1 - \frac{x}{L}(F_2 - F_1) = F_2 - \frac{x}{L}(F_2 - F_1)$

Problem 8. A 10 kg block kept on an inclined plane is pulled by a string applying 200 N force. A 10 N force is also applied on 10 kg block as shown in figure.

Find : (a) tension in the string.

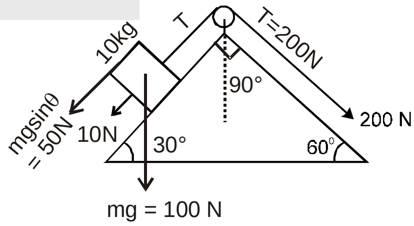
(b) acceleration of 10 kg block.

(c) net force on pulley exerted by string



Answer : (a) 200 N, (b) 14 m/s², (c) $200\sqrt{2}$ N

Solution : (a) T = **200 N**

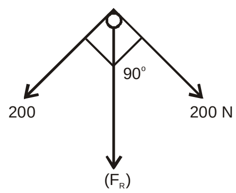


(b) $T - 10 - mg\sin\theta = ma$

$\Rightarrow$ $T - 10 - 50 = 10a$

$\Rightarrow$ $200 - 60 = 10a$

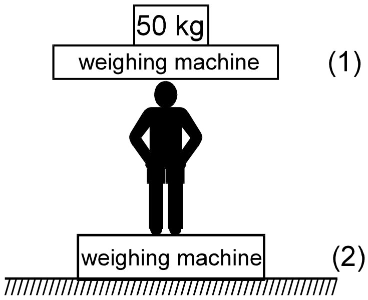
$\Rightarrow$ $a = \frac{140}{10}$ = **14 m/s²**



(c) $(F_R) = \sqrt{(200)^2 + (200)^2}$

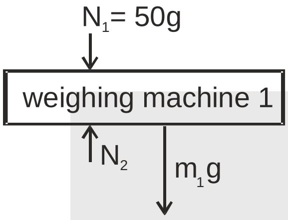
= **$200\sqrt{2}$ N Ans.**

Problem 9. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another similar weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1). Calculate the readings of weighing machines (1) and (2) ($g = 10\ m/s^2$)

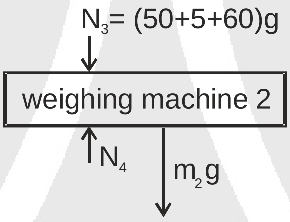


Answer : **500 N, 1150 N**

Solution :

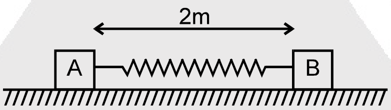


$R_1 = N_1 = 50 \times g = 500$ N
where R_1 = reading in weighing machine 1



$R_2 = N_3 = (50 + 5 + 60)\ g = 115 \times 10 = 1150$ N where R_2 = reading in weighing machine 2

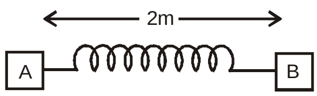
Problem 10. Two blocks are connected by a spring of natural length 2 m.



The force constant of spring is 200 N/m. Find spring force in following situations :
(a) A is kept at rest and B is displaced by 1 m in right direction.
(b) B is kept at rest and A is displaced by 1m in left direction.
(c) A is displaced by 0.75 m in right direction, and B is 0.25 m in left direction.

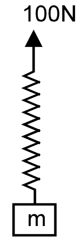
Answer : (a) F = 200 N, (b) 200 N, (c) 200 N

Solution : (a) Extension in spring = 1 m.
$\therefore \quad F_{spring} = Kx = 200 \times 1 = 200$ N



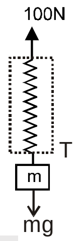
(b) Same extension same spring force in both directions $F_{spring} = 200$ N.
(c) Both displacements of A of B are compressing the spring total compressing $= 0.75 + 0.25 = 1$ m.
$\therefore \quad F_{spring} = kx = 200 \times 1 = 200$ N.

Problem 11. If force constant of spring is 50 N/m. Find mass of the block, if it is at rest in the given situation. ($g = 10\ m/s^2$)



Answer : m = 10 kg

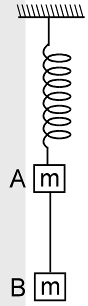
Solution : T = 100 N



$\Rightarrow$ mg = 100 N

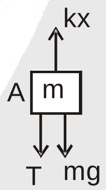
$m = \frac{100}{g} = 10$ kg.

Problem 12. Two blocks 'A' and 'B' of same mass 'm' attached with a light string are suspended by a spring as shown in figure Find the acceleration of block 'A' and 'B' just after the string is cut..



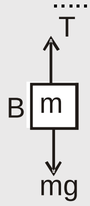
Answer : g (upwards), g (downwards)

Solution : **<u>When string is not cut</u> :**



<u>FBD of 'A' block</u>

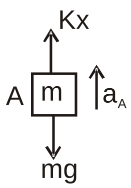
kx = mg + T ......(i)



<u>F. B.D of 'B' block</u>

T = mg ......(ii)

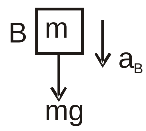
<u>When string is cut :</u>



<u>FBD of 'A' block</u>

$kx - mg = ma_A$

$2mg - mg = ma_A = a_A = g$ (upwards)

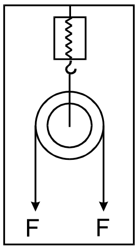


<u>FBD of 'B' block</u>

$ma_B = mg$

$a_B = g$ (downwards)

Problem 13. Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.

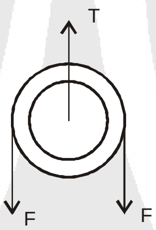


Answer : 2F

Solution : **FBD of spring balance**

R = T .....(i)

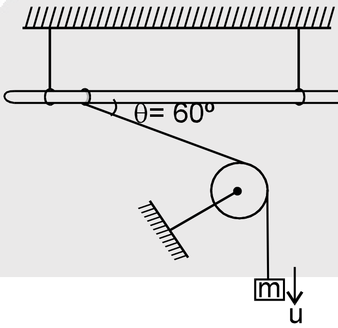
FBD of pulley



T = 2 F .....(ii)

R = 2F

Problem 14. The figure shows mass m moves with velocity u. Find the velocity of ring at that moment. Ring is restricted to move on smooth rod.



Answer : $V_R = \frac{u}{\cos\theta}$, $V_R = 2u$

Solution : Velocity along string remains same .

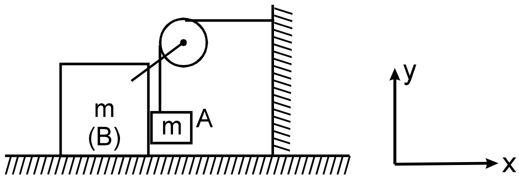
$V_R \cos\theta = u$

$V_R = \frac{u}{\cos\theta}$

$\theta = 60^o$

$V_R = 2u$

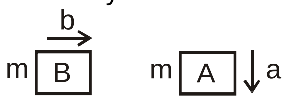
Problem 15. In the system shown in figure, the block A is released from rest. Find :



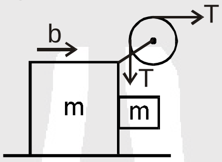
(i) The acceleration of both blocks 'A' and 'B'.
(ii) Tension in the string.
(iii) Contact force between 'A' and 'B'.

Answer : (i) $\frac{g}{3}\hat{i}-\frac{g}{3}\hat{j}$, $\frac{g}{3}\hat{i}$ (ii) $\frac{2mg}{3}$ (iii) $\frac{mg}{3}$

Solution : (i) Let acceleration of blocks in x & y directions are

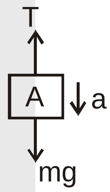


<u>Taking both blocks as a system</u>



T = 2mb .....(i)

<u>Taking A block :</u>



mg – T = ma .....(ii)

From equations (i) & (ii) ; ma + 2 mb = mg

a + 2 b = g .....(iii)

From string constraint ;

a = b .....(iv)

From equations (iii) & (iv);

$a = b = \frac{g}{3}$

Hence, acceleration of block A

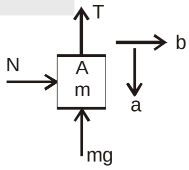
$a_A = b\hat{i} - a\hat{j}$ $\quad a_A = \frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$

Acceleration of block B $a_B = b\hat{i} = \frac{g}{3}\hat{i}$

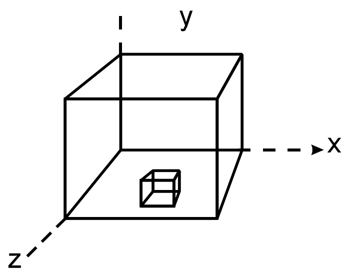
(ii) $T = 2mb = \frac{2mg}{3}$

(iii) For contact force between 'A' and 'B'

<u>FBD of block 'A'</u> : N = mb $\quad N = \frac{mg}{3}$



Problem 16. A block of mass 2 kg is kept at rest on a big box moving with velocity $2\hat{i}$ and having acceleration $-3\hat{i}+4\hat{j}$ m/s^2. Find the value of 'Pseudo force' acting on block with respect to box



Answer : $\vec{F}_{pseudo} = -m\vec{a}_{frame} = -2(-3\hat{i}+4\hat{j})$

$F = 6\hat{i}-8\hat{j}$.

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

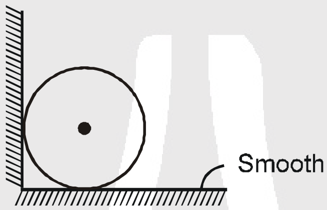
Section (A) : Type of forces, Newton's third law, Free body diagram :

A-1. Which type of forces does a proton exerts on a proton inside nucleus ?

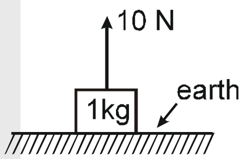
A-2. A block 'A' exerts a force on 'B' of magnitude 20 N. Calculate the magnitude of force exerted by 'B' on 'A'.

A-3. Two forces of same magnitude act on an isolated body in opposite directions to keep it at equilibrium position, is this true according to Newton's third law ?

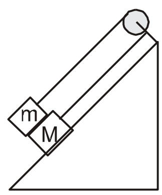
A-4. Draw F.B.D. of the sphere of mass M placed between a vertical wall and ground as shown in figure (All surfaces are smooth).



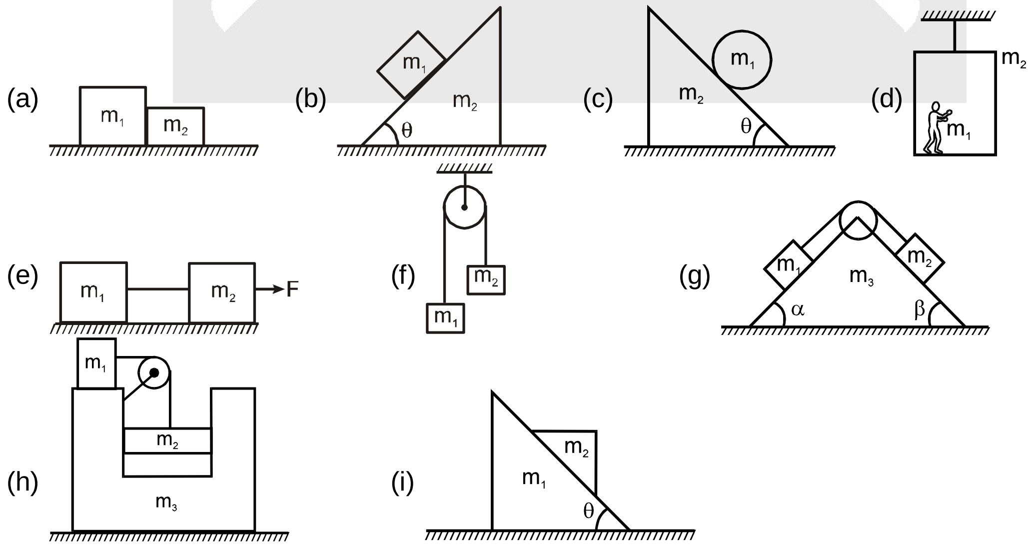
A-5.✍ A block of mass 1 kg placed on ground is pulled by a string by applying 10 N force : ($g = 10\ m/s^2$)
(i) Draw F.B.D. of block.
(ii) Give action–reaction pair involved in the above problem.



A-6. Draw free body diagrams for masses m and M shown in figure. Identify all action-reaction pairs between two blocks. The pulley is frictionless and massless and all surfaces are smooth.

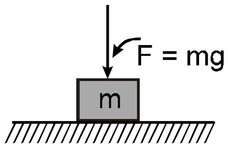


A-7. Draw the FBD for the following individual parts of the systems :
(Pulley are massless and friction less)

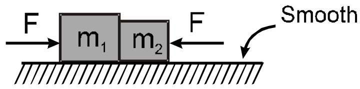


Section (B) : Calculation of normal reaction

B-1. A block of mass 'm' is placed on ground and an additional force F = mg is applied on the block as shown in figure. Calculate contact force between ground and block.

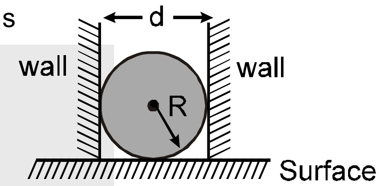


B-2. Two blocks of masses m_1 and m_2 are placed on ground as shown in figure. Two forces of magnitude F act on m_1 and m_2 in opposite directions.

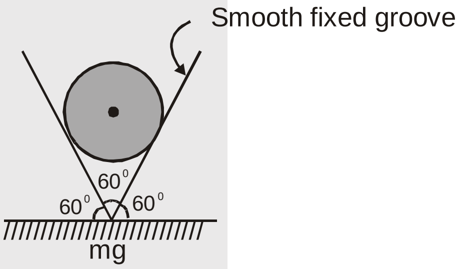


(i) Draw F.B.D. of masses m_1 and m_2.
(ii) Calculate the contact force between m_1 and m_2.
(iii) What will be the value of normal force between m_1 and m_2 .
(iv) Calculate force exerted by ground surface on mass m_1 and m_2

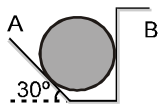
B-3. A sphere of mass 'm', radius 'R' placed between two vertical walls having separation 'd' which is slightly greater than '2R' :
(i) Calculate force exerted by walls on the sphere.
(ii) Calculate force exerted by ground surface on the sphere.



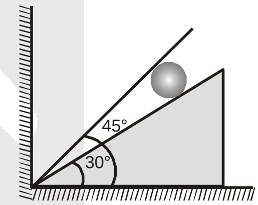
B-4. A cylinder of weight w is resting on a fixed V-groove as shown in figure.
(a) Draw its free body diagram.
(b) Calculate normal reactions between the cylinder and two inclined walls.



B-5. The 50 kg homogeneous smooth sphere rests on the 30º incline A and bears against the smooth vertical wall B. Calculate the contact forces at A and B.



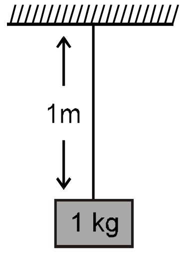
B-6. A spherical ball of mass m = 5 kg rests between two planes which make angles of 30° and 45° respectively with the horizontal. The system is in equilibrium. Find the normal forces exerted on the ball by each of the planes. The planes are smooth.



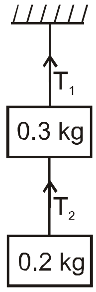
Section (C) : Calculation of Tension

C-1. A one meter long massless string fixed with a wall is pulled horizontally by applying a force of magnitude 10 N. Calculate:
(a) the tension at a point 0.5m away from wall.
(b) the tension at a point 0.75 m away from wall.
(c) force exerted by string on the rigid support.

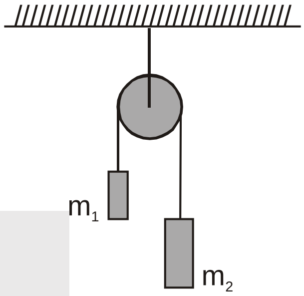
C-2. A block of mass 1 kg is suspended by a string of mass 1 kg, length 1m as shown in figure. ($g = 10\ m/s^2$) Calculate:
(a) the tension in string at its lowest point.
(b) the tension in string at its mid–point.
(c) force exerted by support on string.



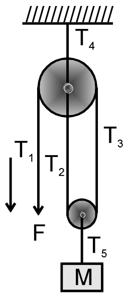
C-3. A block of mass 0.3 kg is suspended from the ceiling by a light string. A second block of mass 0.2 kg is suspended from the first block through another string. Find the tensions in the two strings.
Take g = 10 m/s^2.



C-4. Two unequal masses m_1 and m_2 are connected by a string going over a clamped light smooth pulley as shown in figure m_1 = 3 kg and m_2 = 6 kg. The system is released from rest. (a) Find the distance travelled by the first block in the first two seconds. (b) Find the tension in the string. (c) Find the force exerted by the clamp on the pulley. (g = 10 m/s^2)

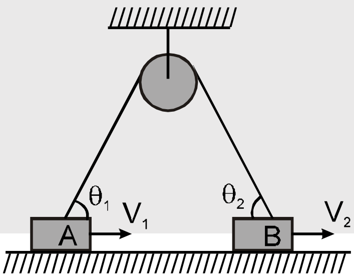


C-5. A mass M is held in place by an applied force F and a pulley system as shown in figure. The pulleys are massless and frictionless.
(a) Draw a free body diagram for each pulley
(b) Find the tension in each section of rope T_1, T_2, T_3, T_4 and T_5.
(c) Find the magnitude of F.

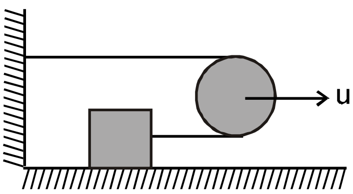


Section (D) : Constrained motion

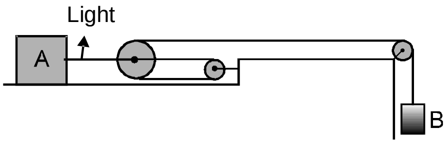
D-1. In the figure shown, blocks A and B move with velocities v_1 and v_2 along horizontal direction. Find the ratio of v_1/v_2.



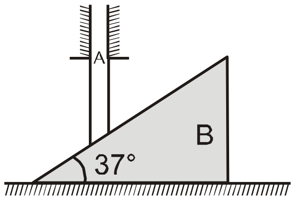
D-2. In the figure shown, the pulley is moving with velocity u. Calculate the velocity of the block attached with string.



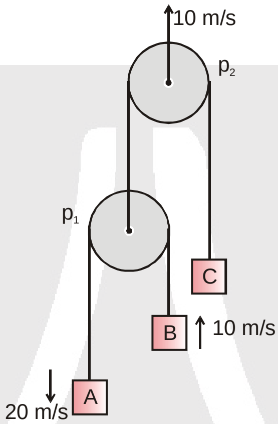
D-3. If block A has a velocity of 0.6 m/s to the right, determine the velocity of block B.



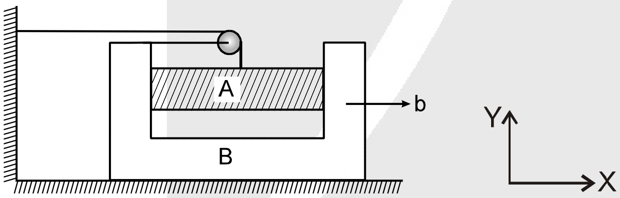
D-4. Find the acceleration of rod A and wedge B in the arrangement shown in figure if the mass of rod equal that of the wedge and the friction between all contact surfaces is negligible and rod A is free to move downwards only. Take angle of wedge as 37°.



D-5. Velocities of blocks A, B and pulley p_2 are shown in figure. Find velocity of pulley p_1 and block C.



D-6. Find the acceleration of A in term of b.

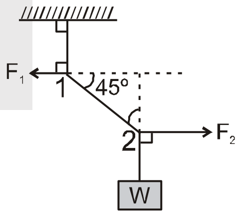


Section (E) : Calculation of Force and Acceleration

E-1. In the figure the tension in the string between 1 and 2 is 60 N.

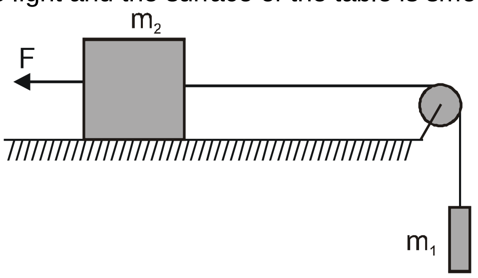
(a) Find the magnitude of the horizontal force $\vec{F}_1$ and $\vec{F}_2$ that must be applied to hold the system in the position shown.

(b) What is the weight of the suspended block ?

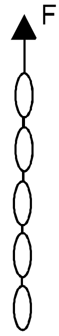


E-2. A 3.0 kg mass is moving in a plane, with its x and y coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where x and y are in meters and t is in second. Find the magnitude of the net force acting on this mass at t = 2 sec.

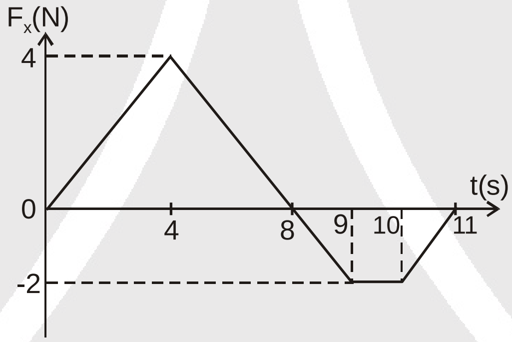
E-3. A constant force $F = m_1g / 2$ is applied on the block of mass m_2 as shown in figure. The string and the pulley are light and the surface of the table is smooth. Find the acceleration of m_2.



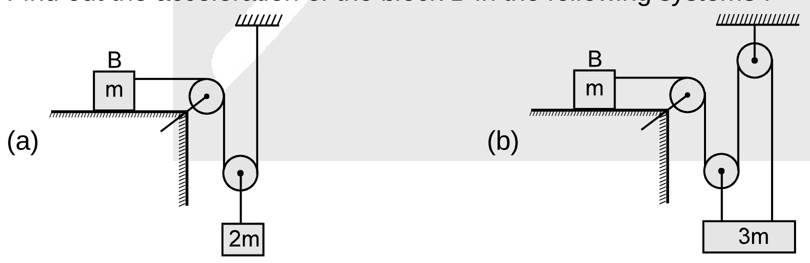
E-4. A chain consisting of five links each with mass 100gm is lifted vertically with constant acceleration of $2 m/s^2$ (↑) as shown. Find : ($g = 10 m/s^2$) :
(a) the forces acting between adjacent links
(b) the force F exerted on the top link by the agent lifting the chain
(c) the net force on each link.



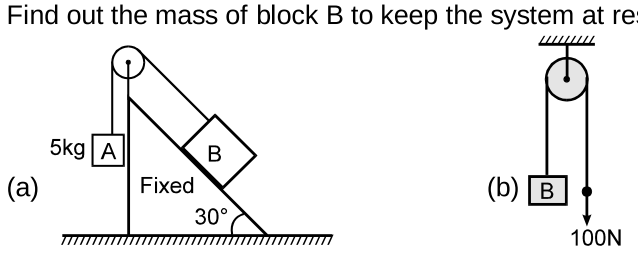
E-5. A 2 kg toy car can move along an x axis. Graph shows force F_x, acting on the car which begins at rest at time t = 0. The velocity of the car at t = 10 s is :



E-6. Find out the acceleration of the block B in the following systems :



E-7. Find out the mass of block B to keep the system at rest : ($g = 10 m/s^2$)

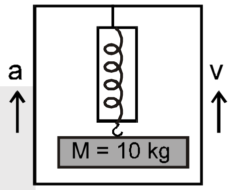


Section (F) : Weighing machine, Spring related problems and Spring balance

F-1. A man of mass 60 kg is standing on a weighing machine placed in a lift moving with velocity 'v' and acceleration 'a' as shown in figure. Calculate the reading of weighing machine in following situations : ($g = 10\ m/s^2$)

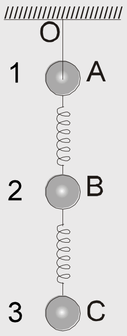
(i) a = 0, v = 0
(ii) a = 0, v = 2 m/s
(iii) a = 0, v = – 2m/s
(iv) $a = 2m/s^2$, v = 0
(v) $a = -2\ m/s^2$ v = 0
(vi) $a = 2\ m/s^2$, v = 2 m/s
(vii) $a = 2\ m/s^2$, v = – 2 m/s
(viii) $a = -2\ m/s^2$, v = – 2 m/s

F-2. What will be the reading of spring balance in the figure shown in following situations? ($g = 10\ m/s^2$)



(i)	a = 0,	v = 0
(ii)	a = 0,	v = 2 m/s
(iii)	a = 0,	v = – 2m/s
(iv)	$a = 2m/s^2$,	v = 0
(v)	$a = -2\ m/s^2$	v = 0
(vi)	$a = 2\ m/s^2$,	v = 2 m/s
(vii)	$a = 2\ m/s^2$,	v = – 2 m/s
(viii)	$a = -2\ m/s^2$	v = – 2 m/s

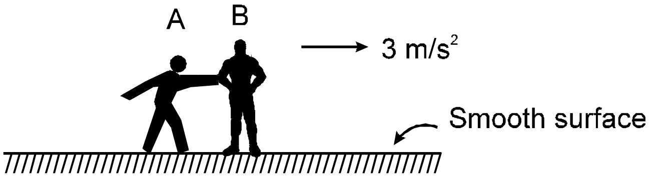
F-3. Three identical balls 1,2,3 are suspended on springs one below the other as shown in the figure. OA is a weightless thread. The balls are in equilibrium



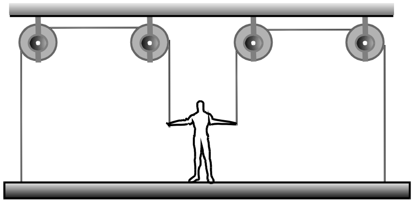
(a) If the thread is cut, the system starts falling. Find the acceleration of all the balls at the initial instant
(b) Find the initial accelerations of all the balls if we cut the spring BC, which is supporting ball 3, instead of cutting the thread.

Section (G) : Newton's law for a system

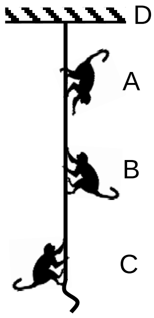
G-1. Man 'A' of mass 60 kg pushes the other man 'B' of mass 75 kg due to which man 'B' starts moving with acceleration $3\ m/s^2$. Calculate the acceleration of man 'A' at that instant.



G-2. A painter of mass M stands on a platform of mass m and pulls himself up by two ropes which hang over pulley as shown. He pulls each rope with the force F and moves upward with uniform acceleration 'a'. Find 'a'



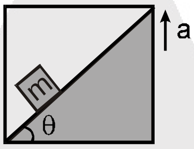
G-3. Three monkeys A, B and C with masses of 10, 15 & 8 Kg respectively are climbing up & down the rope suspended from D. At the instant represented, A is descending the rope with an acceleration of 2 m/s^2 & C is pulling itself up with an acceleration of 1.5 m/s^2. Monkey B is climbing up with a constant speed of 0.8 m/s. Calculate the tension T in the rope at D. (g = 10 m/s^{-2})



Section (H) : Pseudo Force

H-1. An object of mass 2 kg moving with constant velocity $10\hat{i}$ m/s is seen in a frame moving with constant velocity $10\hat{i}$ m/s. What will be the value of 'pseudo force' acting on object in this frame.

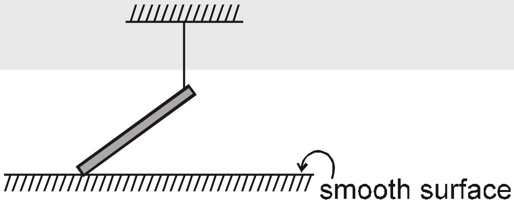
H-2. In the adjoining figure, a wedge is fixed to an elevator moving upwards with an acceleration 'a'. A block of mass 'm' is placed over the wedge. Find the acceleration of the block with respect to wedge. Neglect friction.

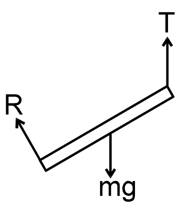


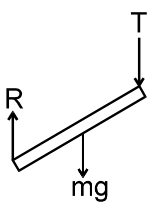
PART - II : ONLY ONE OPTION CORRECT TYPE

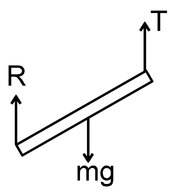
Section (A) : Type of forces, Newton's third law, Free body diagram :

A-1. Which figure represents the correct F.B.D. of rod of mass m as shown in figure :



(A) 

(B) 

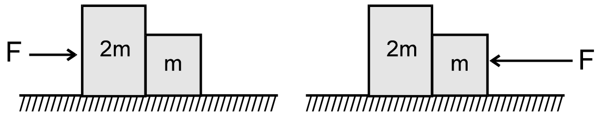
(C) 

(D) None of these

A-2. When a horse pulls a cart , the force needed to move the horse in forward direction is the force exerted by
(A) the cart on the horse (B) the ground on the horse
(C) the ground on the cart (D) the horse on the ground

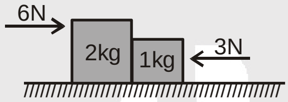
Section (B) : Calculation of normal reaction

B-1. Two blocks are in contact on a frictionless table. One has mass m and the other 2m. A force F is applied on 2m as shown in the figure. Now the same force F is applied from the right on m. In the two cases respectively, the ratio of force of contact between the two blocks will be :



(A) same (B) 1 : 2 (C) 2 : 1 (D) 1 : 3

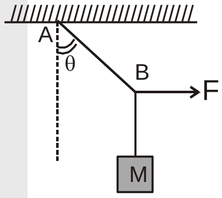
B-2. Two forces of 6N and 3N are acting on the two blocks of 2kg and 1kg kept on frictionless floor. What is the force exerted on 2kg block by 1kg block ?



(A) 1N (B) 2N (C) 4N (D) 5N

Section (C) : Calculation of Tension

C-1. A mass M is suspended by a rope from a rigid support at A as shown in figure. Another rope is tied at the end B, and it is pulled horizontally with a force F. If the rope AB makes an angle θ with the vertical in equilibrium, then the tension in the string AB is :

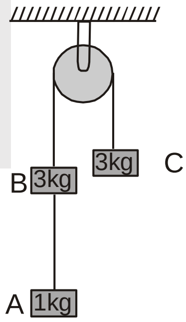


(A) $F \sin \theta$ (B) $F/\sin \theta$
(C) $F \cos \theta$ (D) $F/\cos \theta$

C-2. Two persons are holding a light rope tightly at its ends so that it is horizontal. A 15 kg weight is attached to the rope at the mid point which now no longer remains horizontal. The minimum tension required to completely straighten the rope is :

(A) 15 kg (B) $\frac{15}{2}$ kg (C) 5 kg (D)Infinitely large(or not possible)

C-3. In the system shown in the figure, the acceleration of the 1 kg mass and the tension in the string connecting between A and B is :



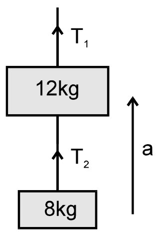
(A) $\frac{g}{4}$ downwards , $\frac{8g}{7}$

(B) $\frac{g}{4}$ upwards, $\frac{g}{7}$

(C) $\frac{g}{7}$ downwards , $\frac{6}{7}g$

(D) $\frac{g}{2}$ upwards , g

C-4. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled by a string with an acceleration of 2.2 m s^{-2}. The tension T_1 and T_2 will be respectively :
(use $g = 9.8 m/s^2$)

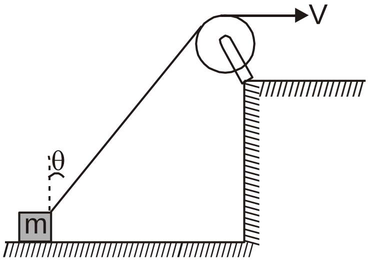


(A) 200 N, 80 N (B) 220 N, 90 N
(C) 240 N, 96 N (D) 260 N, 96 N

C-5. A particle of small mass m is joined to a very heavy body by a light string passing over a light pulley. Both bodies are free to move. The total downward force on the pulley due to string is nearly
(A) mg (B) 2 mg (C) 4 mg (D) can not be determined

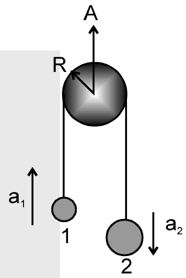
Section (D) : Constrained motion

D-1. A block is dragged on smooth plane with the help of a rope which moves with velocity v. The horizontal velocity of the block is :



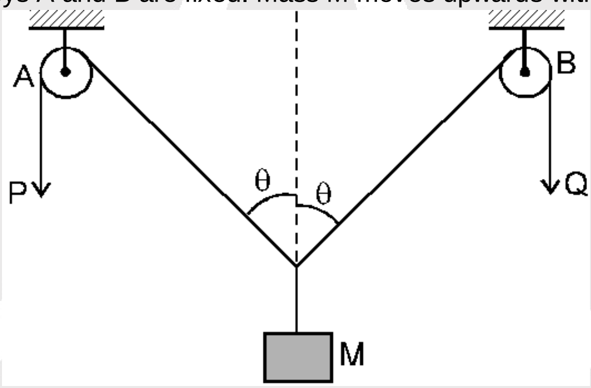
(A) v (B) $\frac{v}{\sin\theta}$
(C) v sin θ (D) $\frac{v}{\cos\theta}$

D-2. Two masses are connected by a string which passes over a pulley accelerating upward at a rate A as shown. If a_1 and a_2 be the acceleration of bodies 1 and 2 respectively then :



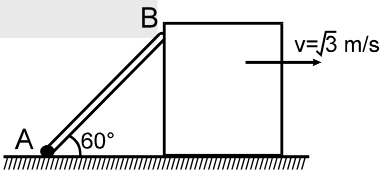
(A) $A = a_1 - a_2$ (B) $A = a_1 + a_2$
(C) $A = \frac{a_1 - a_2}{2}$ (D) $A = \frac{a_1 + a_2}{2}$.

D-3. In the arrangement shown in fig. the ends P and Q of an unstretchable string move downwards with uniform speed U. Pulleys A and B are fixed. Mass M moves upwards with a speed.



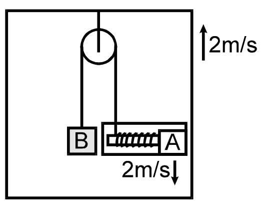
(A) 2 U cos θ (B) U cos θ (C) 2 U/cos θ (D) U/cos θ

D-4. A rod AB is shown in figure. End A of the rod is fixed on the ground. Block is moving with velocity $\sqrt{3}$ m/s towards right. The velocity of end B of rod when rod makes an angle of 60º with the ground is:



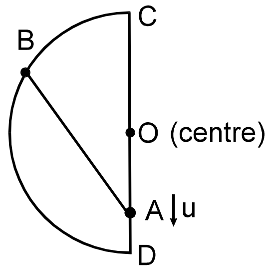
(A) $\sqrt{3}$ m/s (B) 2 m/s
(C) 2 $\sqrt{3}$ m/s (D) 3 m/s

D-5. In the figure shown the velocity of lift is 2 m/s while string is winding on the motor shaft with velocity 2 m/s and block A is moving downwards with a velocity of 2 m/s, then find out the velocity of block B.



(A) 2 m/s ↑ (B) 2 m/s ↓
(C) 4 m/s ↑ (D) 8 m/s ↑

D-6. Two beads A and B move along a fixed semicircular wire frame as shown in figure. The beads are connected by an inelastic string which always remains tight. At an instant the speed of A is u, $\angle BAC = 45°$ and $\angle BOC = 75°$, where O is the centre of the semicircular arc. The speed of bead B at that instant is :



(A) $\sqrt{2}$ u (B) u

(C) $\frac{u}{2\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}$ u

Section (E) : Calculation of Force and Accelertion

E-1. A particle is moving with a constant speed along a straight line path. A force is not required to :

(A) increase its speed (B) decrease its momentum

(C) change the direction (D) keep it moving with uniform velocity

E-2. An object will continue accelerating until :

(A) resultant force on it begins to decrease

(B) its velocity changes direction

(C) the resultant force on it is zero

(D) the resultant force is at right angles to its direction of motion

E-3. In which of the following cases the net force is not zero ?

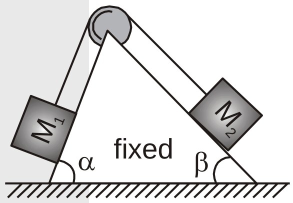
(A) A kite skillfully held stationary in the sky

(B) A ball freely falling from a height

(C) An aeroplane rising upwards at an angle of 45º with the horizontal with a constant speed

(D) A cork lying on the surface of water

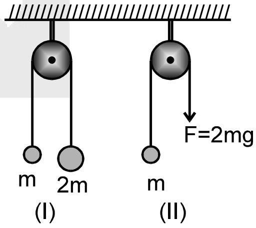
E-4. Two masses M_1 and M_2 are attached to the ends of a light string which passes over a massless pulley attached to the top of a double inclined smooth plane of angles of inclination α and β. If $M_2 > M_1$ and $\beta > \alpha$ then the acceleration of block M_2 down the inclined will be:



(A) $\frac{M_2 g(\sin\beta)}{M_1 + M_2}$ (B) $\frac{M_1 g(\sin\alpha)}{M_1 + M_2}$

(C) $\left(\frac{M_2 \sin\beta - M_1 \sin\alpha}{M_1 + M_2}\right) g$ (D) zero

E-5. The pulley arrangements shown in figure are identical, the mass of the rope being negligible. In case I, the mass m is lifted by attaching a mass 2m to the other end of the rope. In case II, the mass m is lifted by pulling the other end of the rope with a constant downward force F = 2mg, where g is acceleration due to gravity. The acceleration of mass in case I is:



(A) zero (B) more than that in case II

(C) less than that in case II (D) equal to that in case II

E-6. A force produces an acceleration of 4 ms^{-2} in a body of mass m_1 and the same force produces an acceleration of 6 ms^{-2} in another body of mass m_2. If the same force is applied to ($m_1 + m_2$), then the acceleration will be:

(A) 10 ms^{-2} (B) 2 ms^{-2} (C) 2.4 ms^{-2} (D) 5.4 ms^{-2}

E-7. A body of mass M is acted upon by a force F and the acceleration produced is a. If three coplaner forces each equal to F and inclined to each other at 120º act on the same body and no other forces are acting. The acceleration produced will be:

(A) $\sqrt{2}$ a (B) a/ $\sqrt{3}$ (C) 3a (D) zero

E-8. A fireman wants to slide down a rope. The rope can bear a tension of $\frac{3}{4}^{th}$ of the weight of the man. With what minimum acceleration should the fireman slide down :

(A) $\frac{g}{3}$ (B) $\frac{g}{6}$ (C) $\frac{g}{4}$ (D) $\frac{g}{2}$

E-9. A force $\vec{F} = 6\hat{i} - 8\hat{j} + 10\hat{k}$ newton produces acceleration 1 m/s^2 in a body. The mass of the body is (in kg) :

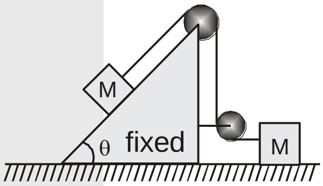
(A) $6\hat{i} - 8\hat{j} + 10\hat{k}$ kg (B) $10\sqrt{2}$ kg

(C) 100 kg (D) 10 kg

E-10. A body is moving with a speed of 1 m/s and a constant force F is needed to stop it in a distance x. If the speed of the body is 3 m/s the force needed to stop it in the same distance x will be

(A) 1.5 F (B) 3F (C) 6 F (D) 9F

E-11. Two blocks, each having mass M, rest on frictionless surfaces as shown in the figure. If the pulleys are light and frictionless, and M on the incline is allowed to move down, then the tension in the string will be:

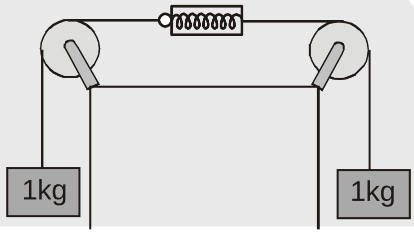


(A) $\frac{2}{3}$ Mg sin θ (B) $\frac{3}{2}$ Mg sin θ

(C) $\frac{Mg\sin\theta}{2}$ (D) 2 Mg sin θ

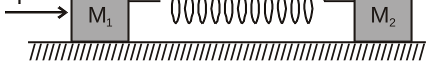
Section (F) : Weighing machine, Spring related problems and Spring balance

F-1. In the given figure, what is the reading of the spring balance ?



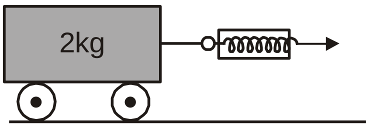
(A) 10 N (B) 20 N (C) 5 N (D) zero

F-2. Two blocks of masses M_1 and M_2 are connected to each other through a light spring as shown in figure. If we push mass M_1 with force F and cause acceleration a_1 in right direction. What will be the magnitude of acceleration in M_2?



(A) F/M_2 (B) $F/(M_1 + M_2)$ (C) a_1 (D) $(F - M_1a_1)/M_2$

F-3. A massless spring balance is attached to 2 kg trolley and is used to pull the trolley along a flat surface as shown in the fig. The reading on the spring balance remains at 10 kg during the motion. The acceleration of the trolley is (Use g = 9.8 ms^{-2})



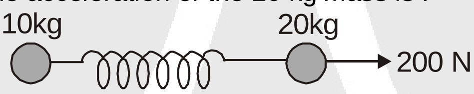
(A) 4.9 ms^{-2} (B) 9.8 ms^{-2} (C) 49 ms^{-2} (D) 98 ms^{-2}

F-4. The ratio of the weight of a man in a stationary lift & when it is moving downward with uniform acceleration 'a' is 3 : 2 . The value of 'a' is : (g = acceleration. due to gravity)
(A) (3/2) g (B) g (C) (2/3) g (D) g/3

F-5. A body of mass 32 kg is suspended by a spring balance from the roof of a vertically operating lift and going downward from rest. At the instants the lift has covered 20 m and 50 m, the spring balance showed 30 kg & 36 kg respectively. The velocity of the lift is:
(A) Decreasing at 20 m & increasing at 50 m
(B) increasing at 20 m & decreasing at 50 m
(C) Continuously decreasing at a constant rate throughout the journey
(D) Continuously increasing at constant rate throughout the journey
(E) Remaining constant throughout the journey.

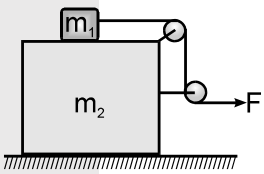
Section (G) : Newton's law for a system

G-1. Two masses of 10 kg and 20 kg respectively are connected by a massless spring as shown in figure. A force of 200 N acts on the 20 kg mass at the instant when the 10 kg mass has an acceleration of 12 ms^{-2} towards right, the acceleration of the 20 kg mass is :



(A) 2 ms^{-2} (B) 4ms^{-2} (C) 10ms^{-2} (D) 20ms^{-2}

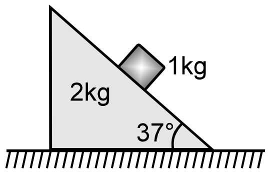
G-2. In the arrangement shown in the figure all surfaces are frictionless, pulley and string are light. The masses of the block are m_1 = 20 kg and m_2 = 30 kg. The accelerations of masses m_1 and m_2 will be if F = 180 N is applied according to figure.



(A) $a_{m_1} = 9 m/s^2$, $a_{m_2} = 0$ (B) $a_{m_1} = 9 m/s^2$, $a_{m_2} = 9$ m/s2
(C) $a_{m_1} = 0$, $a_{m_2} = 9$ m/s^2 (D) None of these

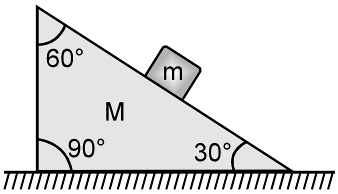
Section (H) : Pseudo force

H-1. Figure shows a wedge of mass 2kg resting on a frictionless floor. A block of mass 1 kg is kept on the wedge and the wedge is given an acceleration of 5 m/sec^2 towards right. Then :



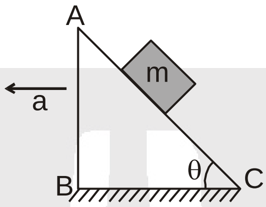
(A) block will remain stationary w.r.t. wedge
(B) the block will have an acceleration of 1 m/sec^2 w.r.t. the wedge
(C) normal reaction on the block is 11 N
(D) net force acting on the wedge is 2 N

H-2. A triangular block of mass M rests on a smooth surface as shown in figure. A cubical block of mass m rests on the inclined surface. If all surfaces are frictionless, the force that must be applied to M so as to keep m stationary relative to M is :



(A) Mg tan 30° (B) mg tan 30° (C) (M+m)g tan 30° (D) (M+m)g cos 30°

H-3. A block of mass m resting on a wedge of angle θ as shown in the figure. The wedge is given an acceleration a towards left. What is the minimum value of **a** due to external agent so that the mass m falls freely ?

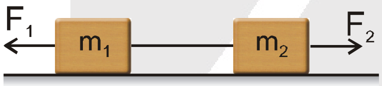


(A) g (B) g cos θ (C) g cot θ (D) g tan θ

PART - III : MATCH THE COLUMN

1. Column-I gives four different situations involving two blocks of mass m_1 and m_2 placed in different ways on a smooth horizontal surface as shown. In each of the situations horizontal forces F_1 and F_2 are applied on blocks of mass m_1 and m_2 respectively and also $m_2 F_1 < m_1 F_2$. Match the statements in column I with corresponding results in column-II

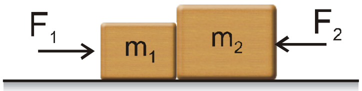
Column I

(A) 

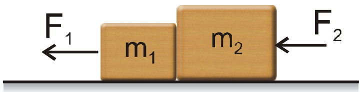
. Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is

(B) 

. Both the blocks are connected by massless inelastic string. The magnitude of tension in the string is

(C) 

. The magnitude of normal reaction between the blocks is

(D) 

. The magnitude of normal reaction between the blocks is

Column II

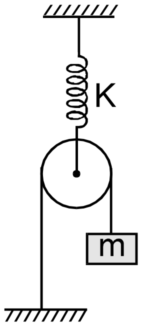
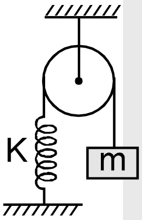
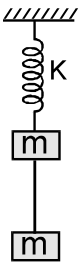
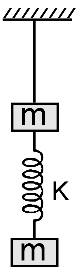
(p) $\frac{m_1 \quad m_2}{m_1+m_2}\left(\frac{F_1}{m_1}-\frac{F_2}{m_2}\right)$

(q) $\frac{m_1 \quad m_2}{m_1+m_2}\left(\frac{F_1}{m_1}+\frac{F_2}{m_2}\right)$

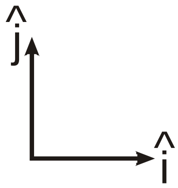
(r) $\frac{m_1 \quad m_2}{m_1+m_2}\left(\frac{F_2}{m_2}-\frac{F_1}{m_1}\right)$

(s) $m_1 \quad m_2 \left(\frac{F_1+F_2}{m_1+m_2}\right)$

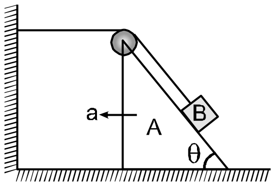
2. In column I four different sitiuation are given and in column II tension in string (which is not connected with spring) & extention in spring at equilibrium is given Match the statements in column I with corresponding results in column-II

Column-I	**Column-II**
(A) 	(p) mg
(B) 	(q) 2mg
(C) 	(r) $\frac{mg}{k}$
(D) 	(s) $\frac{2mg}{k}$

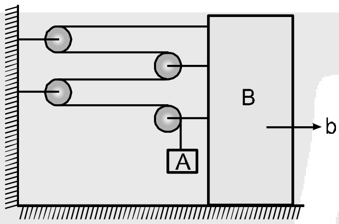
3. In column - I four different situation are given column -II corresponding result are given. Match the statements in column I with corresponding results in column-II in the question below take the unit vector as follows :



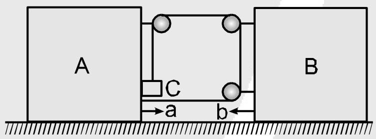
Column– I		**Column– II**
(A)	The acceleration of B w.r.t ground if $\theta = 60°$ & $a = 2m/s^2$	(p) $4\hat{j}$



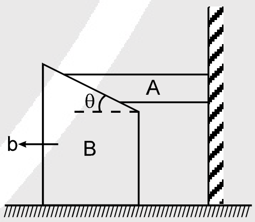
(B).	The acceleration of A w.r.t. ground if $b = 1m/s^2$	(q) $-2\hat{j}$



(C).	The acceleration of C w.r.t. ground if $a = b = 1m/s^2$	(r) $-\hat{i} - \sqrt{3}\hat{j}$



(D)	The acceleration of wedge A if $\theta = 45°$ & $b = 2m/s^2$	(s) $\hat{i} - 4\hat{j}$

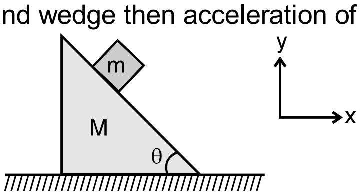


Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

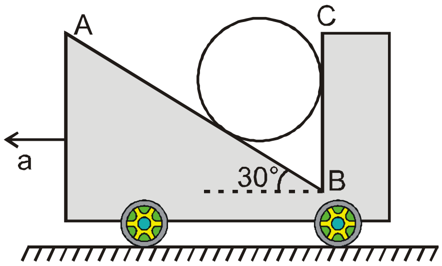
1. Consider the shown arrangement. Assume all surfaces to be smooth. If 'N' represents magnitude of normal reaction between block and wedge then acceleration of 'M' along horizontal equals:



(A) $\frac{N\sin\theta}{M}$ along + ve-x-axis (B) $\frac{N\cos\theta}{M}$ along-ve x-axis

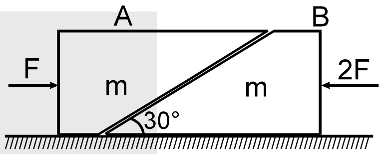
(C) $\frac{N\sin\theta}{M}$ along – ve x-axis (D) $\frac{N\sin\theta}{m+M}$ along –ve x-axis

2. A cylinder rests in a supporting carriage as shown. The side AB of carriage makes an angle 30° with the horizontal and side BC is vertical. The carriage lies on a fixed horizontal surface and is being pulled towards left with an horizontal acceleration '*a*'. The magnitude of normal reactions exerted by sides AB and BC of carriage on the cylinder be N_{AB} and N_{BC} respectively. Neglect friction everywhere. Then as the magnitude of acceleration '*a*' of the carriage is increased, pick up the correct statement:



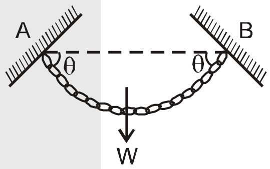
(A) N_{AB} increases and N_{BC} decreases.
(B) Both N_{AB} and N_{BC} increase.
(C) N_{AB} remains constant and N_{BC} increases.
(D) N_{AB} increases and N_{BC} remains constant.

3. Two blocks 'A' and 'B' each of mass 'm' are placed on a smooth horizontal surface. Two horizontal forces F and 2F are applied on the two blocks 'A' and 'B' respectively as shown in figure. The block A does not slide on block B. Then the normal reaction acting between the two blocks is :



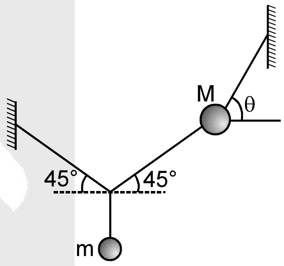
(A) F
(B) F/2
(C) $\frac{F}{\sqrt{3}}$
(D) 3F

4. A flexible chain of weight W hangs between two fixed points A and B at the same level. The inclination of the chain with the horizontal at the two points of support is θ. What is the tension of the chain at the endpoint.



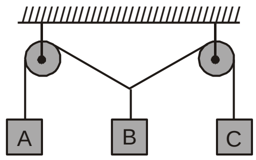
(A) $\frac{W}{2}$ cosec θ
(B) $\frac{W}{2}$ sec θ
(C) W cos θ
(D) $\frac{W}{3}$ sin θ

5. Two masses m and M are attached with strings as shown. For the system to be in equilibrium we have



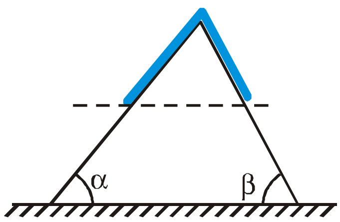
(A) $\tan\theta = 1 + \frac{2M}{m}$
(B) $\tan\theta = 1 + \frac{2m}{M}$
(C) $\tan\theta = 1 + \frac{M}{2m}$
(D) $\tan\theta = 1 + \frac{m}{2M}$

6. Three blocks A , B and C are suspended as shown in the figure. Mass of each block A and C is m. If system is in equilibrium and mass of B is M, then :



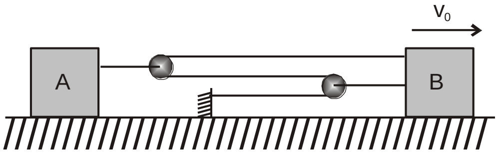
(A) M = 2 m
(B) M < 2 m
(C) M > 2 m
(D) M = m

7. A uniform rope of length L and mass M is placed on a smooth fixed wedge as shown. Both ends of rope are at same horizontal level. The rope is initially released from rest, then the magnitude of initial acceleration of rope is



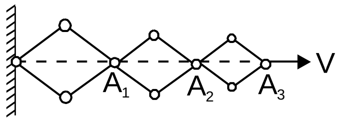
(A) Zero
(B) M(cosα – cos β) g
(C) M(tan α – tan β)g
(D) None of these

8. Block B moves to the right with a constant velocity v_0. The velocity of block A relative to B is :



(A) $\frac{v_0}{2}$, towards left (B) $\frac{v_0}{2}$, towards right (C) $\frac{3v_0}{2}$, towards left (D) $\frac{3v_0}{2}$, towards right

9. A hinged construction consists of three rhombus with the ratio of sides (5 : 3 : 2). Vertex A_3 moves in the horizontal direction with velocity V. Velocity of A_2 will be :

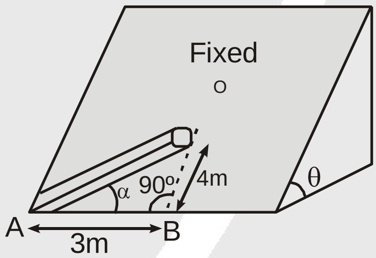


(A) 2.5 V (B) 1.5V (C) (2/3)V (D) 0.8V

10. A balloon of gross weight w newton is falling vertically downward with a constant acceleration a(<g). The magnitude of the air resistance is : (Neglecting buoyant force)

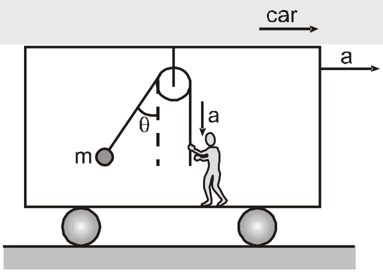
(A) w (B) $w\left(1+\frac{a}{g}\right)$ (C) $w\left(1-\frac{a}{g}\right)$ (D) $w\frac{a}{g}$

11. There is an inclined surface of inclination θ = 30º. A smooth groove is cut into it forming angle α with AB. A steel ball is free to slide along the groove. If the ball is released from the point O at top end of the groove, the speed when it comes to A is: [g = 10 m/s^2]



(A) $\sqrt{40}$m/s (B) $\sqrt{20}$ m/s (C) $\sqrt{10}$ m/s (D) $\sqrt{15}$ m/s

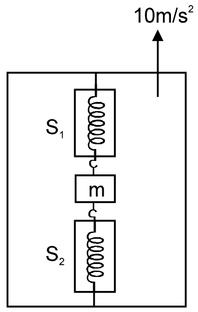
12. A bob is hanging over a pulley inside a car through a string. The second end of the string is in the hand of a person standing in the car. The car is moving with constant acceleration 'a' directed horizontally as shown in figure. Other end of the string is pulled with constant acceleration 'a' (relative to car) vertically. The tension in the string is equal to (assume θ remains constant)



(A) $m\sqrt{g^2+a^2}$ (B) $m\sqrt{g^2+a^2}$ – ma

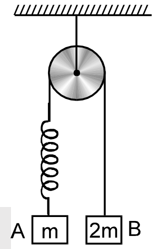
(C) $m\sqrt{g^2+a^2}$ + ma (D) m(g + a)

13. Reading shown in two spring balances S_1 and S_2 is 90 kg and 30 kg respectively when lift is accelerating upwards with acceleration 10 m/s^2. The mass is stationary with respect to lift. Then the mass of the block will be :



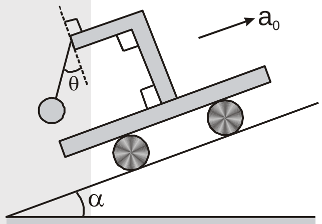
(A) 60 kg
(B) 30 kg
(C) 120 kg
(D) None of these

14. In the figure a block 'A' of mass 'm' is attached at one end of a light spring and the other end of the spring is connected to another block 'B' of mass 2m through a light string. 'A' is held and B is in static equilibrium. Now A is released. The acceleration of A just after that instant is 'a'. In the next case, B is held and A is in static equilibrium. Now when B is released, its acceleration immediately after the release is 'b'. The value of a/b is : (Pulley, string and the spring are massless)



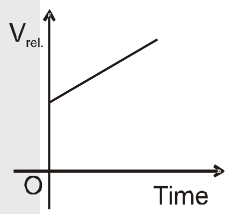
(A) 0 (B) $\frac{1}{2}$ (C) 2 (D) undefined

15. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is (String and bob remain fixed with respect to trolley) :



(A) $\theta = \tan^{-1}\alpha$
(B) $\theta = \tan^{-1}\left(\frac{a_0}{g}\right)$
(C) $\theta = \tan^{-1}$
(D) $\theta = \tan^{-1}\left(\frac{a_0 + g\sin\alpha}{g\cos\alpha}\right)$

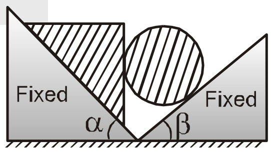
16. A particle is observed from two frames S_1 and S_2. The graph of relative velocity of S_1 with respect to S_2 is shown in figure. Let F_1 and F_2 be the pseudo forces on the particle when seen from S_1 and S_2 respectively. Which one of the following is not possible?



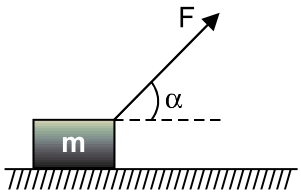
(A) $F_1 = 0, F_2 \neq 0$
(B) $F_1 \neq 0, F_2 = 0$
(C) $F_1 \neq 0, F_2 \neq 0$
(D) $F_1 = 0, F_2 = 0$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

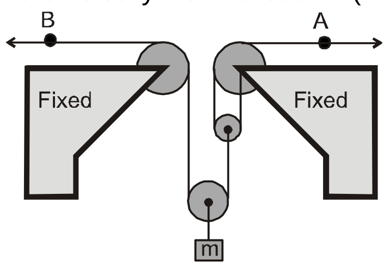
1. A cylinder and a wedge of same masses with a vertical face, touching each other, move along two smooth inclined planes forming the some angle and β respectively with the horizontal. Determine the force of normal N (in newton) exerted by the wedge on the cylinder, neglecting the friction between them. If $m = \frac{1}{\sqrt{3}}$ kg, $\alpha = 60°$, $\beta = 30°$ and $g = 10$m/s^2



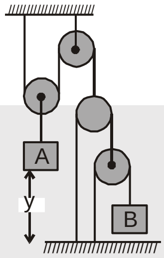
2. At the moment t = 0 the force F = at N is applied to a small body of mass m kg resting on a smooth horizontal plane (a is constant). The permanent direction of this force forms an angle α with the horizontal (as shown in the figure). Then the distance traversed by the body up to this moment of its breaking off the plane is $\frac{m^2 g^3 \cos\alpha}{Pa^2 \sin^3\alpha}$ m. Then find value of P.



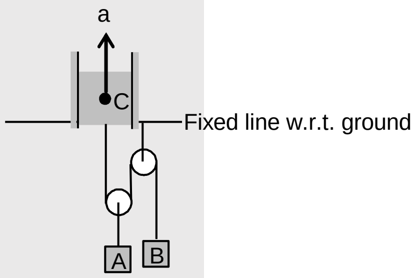
3. For the pulley system, each of the cables at A and B is given velocity of 4m/s in the direction of the arrow. Determine the upward velocity v of the load m. (in m/s)



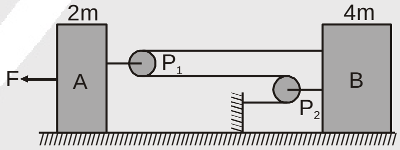
4. The vertical displacement of block A in meter is given by $y = t^2/4$ where t is in second. Calculate the downward acceleration a_B of block B. (in m/s^2)



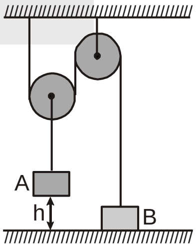
5. The block C shown in the figure is ascending with an acceleration $a = 3$ m/s^2 by means of some motor not shown here. The bodies A and B of masses 10 kg and 5 kg respectively, assuming pulleys and strings are massless and friction is absent everywhere. Then find acceleration of body A. (in m/s^2)



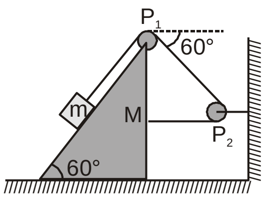
6. The acceleration of the block B in the figure, assuming the surfaces and the pulleys P_1 and P_2 are all smooth and pulleys and string are light is $\frac{3F}{xm}$ then value of x is.



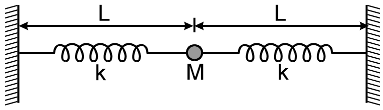
7. In the arrangement shown in figure, the mass of the body A is n = 4 times that of body B. The height h=20 cm. At a certain instant, the body B is released and the system is set in motion. What is the maximum height (in cm) the body B will go up? Assume enough space above B and A sticks to ground. (A and B are of small size) (g = 10 m/s^2)



8. In the arrangement shown in the fig, the block of mass m = 2 kg lies on the wedge of mass M = 8 kg. The initial acceleration of the wedge if the surfaces are smooth and pulley & strings are massless is $\frac{30\sqrt{3}}{x}$ m/s^2 then x is.

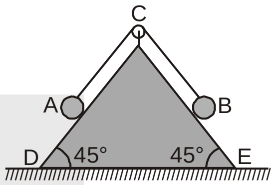


9. A ball of mass M is suspended from two identical springs each with spring constant k and undeformed length L. The ball is held in line with two springs as shown in the figure. When the ball begins to fall, find the magnitude of the acceleration of the ball at the instant when it has fallen through a vertical distance x (in m/s^2) if M = 250g , K = 130N/m , L = 12cm, x= 5cm and g = $10m/s^2$



10. A bead of mass m is fitted on to a rod of a length of 2ℓ and can move on it without friction. At the initial moment the bead is in the middle of the rod. The rod moves translationally in a horizontal plane with an acceleration 'a' in a direction forming an angle α with the rod. Find the time when the bead will leave the rod. If ℓ = 2m, a = $2m/s^2$ and α = 60°

11. Two particles A and B of masses 3 kg and 2 kg are connected by a light inextensible string. The particles are in contact with the smooth faces of a wedge DCE of mass 10 kg resting on a smooth horizontal plane. When the system is moving freely, find the acceleration of the wedge (in cm/s^2).



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

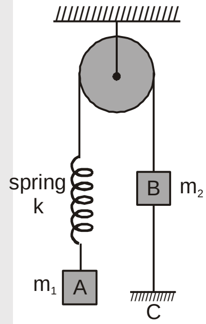
1. In the system shown in the figure $m_1 > m_2$. System is held at rest by thread BC. Just after the thread BC is burnt :

(A) acceleration of m_2 will be upwards

(B) magnitude of acceleration of both blocks will be equal to $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)g$

(C) acceleration of m_1 will be equal to zero

(D) magnitude of acceleration of two blocks will be non–zero and unequal.



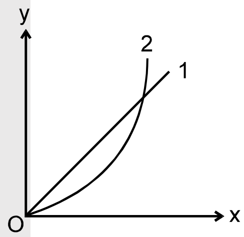
2. A particle is resting on a smooth horizontal floor. At t = 0, a horizontal force starts acting on it. Magnitude of the force increases with time according to law F = α.t, where α is a constant. For the figure shown which of the following statements is/are correct ?

(A) Curve 1 shows acceleration against time

(B) Curve 2 shows velocity against time

(C) Curve 2 shows velocity against acceleration

(D) None of these



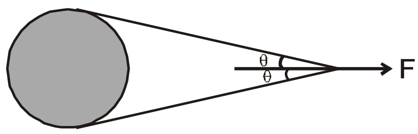
3. A light string is wrapped round a cylindrical log of wood which is placed on a horizontal surface with it's axis vertical and it is pulled with a constant force F as shown in the figure.(Friction is absent everywhere)

(A) tension T in the string increases with increase in θ

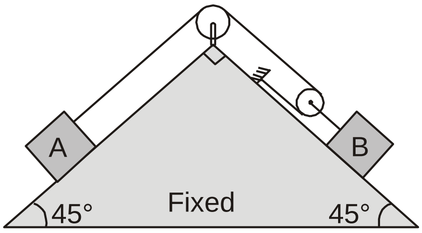
(B) tension T in the string decreases with increase in θ

(C) tension T > F if $\theta > \pi/3$

(D) tension T > F if $\theta > \pi/4$



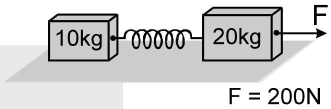
4. Two blocks A and B of mass 10 kg and 40 kg are connected by an ideal string as shown in the figure. Neglect the masses of the pulleys and effect of friction. ($g = 10\ m/s^2$)



(A) The acceleration of block A is $\frac{5}{\sqrt{2}}\ ms^{-2}$ (B) The acceleration of block B is $\frac{5}{2\sqrt{2}}\ ms^{-2}$

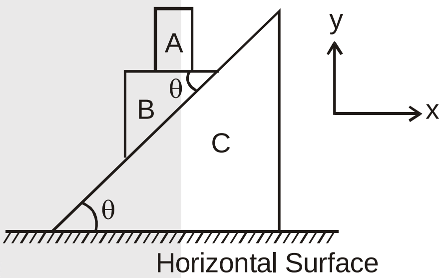
(C) The tension in the string is $\frac{125}{\sqrt{2}}$ N (D) The tension in the string is $\frac{150}{\sqrt{2}}$ N

5. Two blocks of masses 10 kg and 20 kg are connected by a light spring as shown. A force of 200 N acts on the 20 kg mass as shown. At a certain instant the acceleration of 10 kg mass is 12 ms^{-2} towards right direction.



(A) At that instant the 20 kg mass has an acceleration of 12 ms^{-2}.
(B) At that instant the 20 kg mass has an acceleration of 4 ms^{-2}.
(C) The stretching force in the spring is 120 N.
(D) None of these

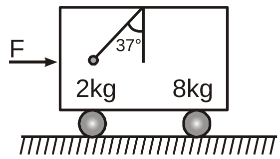
6. In the figure shown all the surface are smooth. All the blocks A, B and C are movable X-axis is horizontal and y-axis vertical as shown. Just after the system is relased from the position as shown.



(A) Acceleration of 'A' relative to ground is in negative y-direction
(B) Acceleration of 'A' relative to B is in positive x-direction
(C) The horizontal acceleration of 'B' relative to ground is in negative x-direction.
(D) The acceleration of 'B' relative to ground directed along the inclined surface of 'C' is greater than $g \sin \theta$.

7. A particle stays at rest as seen from a frame. We can conclude that
(A) the frame is inertial.
(B) resultant force on the particle is zero.
(C) if the frame is inertial then the resultant force on the particle is zero.
(D) if the frame is noninertial then there is a nonzero resultant force.

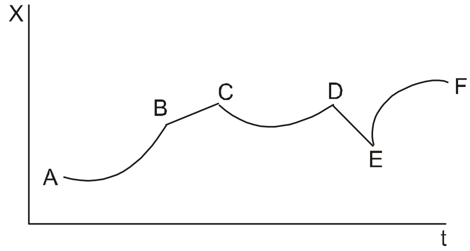
8. A trolley of mass 8 kg is standing on a frictionless surface inside which an object of mass 2 kg is suspended. A constant force F starts acting on the trolley as a result of which the string stood at an angle of 37^0 from the vertical (bob at rest relative to trolley) Then :



(A) acceleration of the trolley is 40/3 m/sec^2.
(B) force applied is 60 N
(C) force applied is 75 N
(D) tension in the string is 25 N

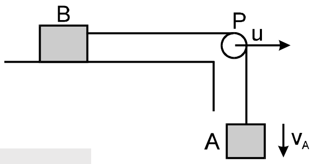
9. A particle is found to be at rest when seen from a frame S_1 and moving with a constant velocity when seen from another frame S_2. Markout the possible options.
(A) Both the frames are inertial (B) Both the frames are noninertial.
(C) S_1 is inertial and S_2 is noninertial. (D) S_1 is noninertial and S_2 is inertial.

10. Figure shows the displacement of a particle going along the X-axis as a function of time. Find the region where force acting on the particle is zero



(A) AB (B) BC (C) CD (D) DE

11. In the Figure, the pulley P moves to the right with a constant speed u. The downward speed of A is v_A, and the speed of B to the right is v_B.



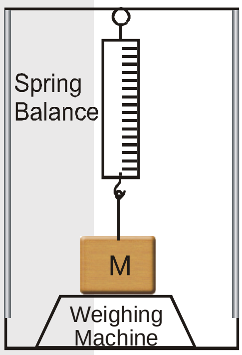
(A) $v_B = v_A$
(B) $v_B = u + v_A$
(C) $v_B + u = v_A$
(D) the two blocks have accelerations of the same magnitude

PART - IV : COMPREHENSION

Comprehension

Figure shows a weighing machine kept in a lift. Lift is moving upwards with acceleration of 5 m/s^2. A block is kept on the weighing machine. Upper surface of block is attached with a spring balance. Reading shown by weighing machine and spring balance is 15 kg and 45 kg respectively.

Answer the following questions. Asume that the weighing machine can measure weight by having negligible deformation due to block, while the spring balance requires larger expansion: (take g = 10 m/s^2)



1. Mass of the object in kg and the normal force acting on the block due to weighing machine are :
(A) 60 kg, 450 N (B) 40 kg, 150 N (C) 80 kg, 400 N (D) 10 kg, zero

2. If lift is stopped and equilibrium is reached. Reading of weighing machine and spring balance will be :
(A) 40 kg, zero (B) 10 kg, 20 kg (C) 20 kg, 10 kg (D) zero, 40 kg

3. Find the acceleration of the lift such that the weighing machine shows true weight of block.
(A) $\frac{45}{4}$ m/s^2 (B) $\frac{85}{4}$ m/s^2 (C) $\frac{22}{4}$ m/s^2 (D) $\frac{60}{4}$ m/s^2

Comprehension 2

A small block of mass 1 kg starts moving with constant velocity 2 m/s on a smooth long plank of mass 10 kg which is also pulled by a horizontal force F = 10 t N where t is in seconds and F is in newtons. (the initial velocity of the plank is zero).

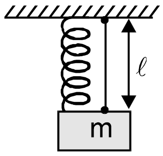
4. Displacement of 1 kg block with respect to plank at the instant when both have same velocity is
(A) $4\frac{4}{3}$ m (B) 4 m (C) $\frac{8}{3}$ m (D) 2 m

5. The time ($t \neq 0$) at which displacement of block and plank with respect to ground is same will be :
(A) 12 s (B) $2\sqrt{3}$ s (C) $3\sqrt{3}$ s (D) $\sqrt{3}/2$ s

6. Relative velocity of plank with respect to block when acceleration of plank is 4 m/s^2 will be
(A) Zero (B) 10 m/s (C) 6 m/s (D) 8 m/s

Comprehension 3

An object of mass m is suspended in equilibrium using a string of length ℓ and a spring having spring constant K (< 2 mg/ℓ) and unstreched length $\ell/2$.



7. Find the tension in the string in newton ?
(A) $mg - \frac{k\ell}{2}$ (B) $mg - k\ell$ (C) $2mg - \frac{k\ell}{2}$ (D) $2mg - k\ell$

8. Find the acceleration of block just after cut the string ?
(A) $2g - \frac{k\ell}{2m}$ (B) $g - \frac{k\ell}{2m}$ (C) $2g - \frac{k\ell}{m}$ (D) $g - \frac{k\ell}{m}$

9. What happens if K > 2 mg/ℓ ?
(A) at equilibrium tension in string is negative
(B) at equilibrium position change in length of spring is greater then $\frac{\ell}{2}$
(C) at equilibrium tension in string is zero.
(D) If we cut the sting, block will accelerate in upword direction.

Exercise-3

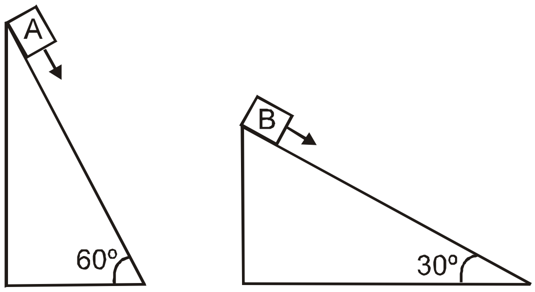
Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. A piece of wire is bent in the shape of a parabola $y = kx^2$ (y-axis vertical) with a bead of mass m on it. The bead can slide on the wire without friction. It stays at the lowest point of the parabola when the wire is at rest. The wire is now accelerated parallel to the x-axis with a constant acceleration a. The distance of the new equilibrium position of the bead, where the bead can stay at rest with respect to the wire, from the y-axis is **[JEE 2009, 3/160, –1]**
(A) $\frac{a}{gk}$ (B) $\frac{a}{2gk}$ (C) $\frac{2a}{gk}$ (D) $\frac{a}{4gk}$

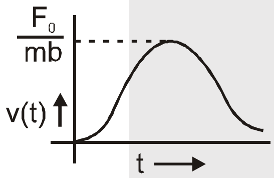
PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

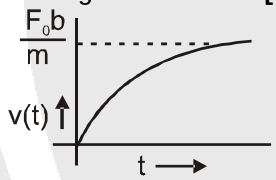
1. Two fixed frictionless inclined planes making an angle 30º and 60º with the vertical are shown in the figure. Two blocks A and B are placed on the two planes respectively. What is the relative vertical acceleration of A with respect to B? **[AIEEE-2010, 4/144, –1]**

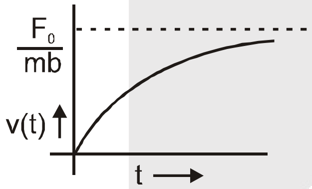


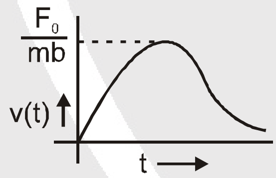
(1) 4.9 ms^{-2} in horizontal direction
(2) 9.8 ms^{-2} in vertical direction
(3) Zero
(4) 4.9 ms^{-2} in vertical direction

2. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves ? **[AIEEE 2012 ; 4/120, –1]**

(1) 

(2) 

(3) 

(4) 

3. A ball is thrown upward with an initial velocity V_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma\upsilon^2$ (where m is mass of the ball, υ is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is : **[JEE (Main) 2019 April ; 4/120, –1]**

(1) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$
(2) $\frac{1}{\sqrt{\gamma g}}\ln\left(1+\sqrt{\frac{\gamma}{g}}V_0\right)$
(3) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}V_0\right)$
(4) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}V_0\right)$

4. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals : (Take g = 10 ms^{-2} and the rope to be massless)
[JEE (Main) 2020, 07 January; 4/100, –1]

(1) 75 N (2) 90 N (3) 100 N (4) 70 N

Answers

EXERCISE-1

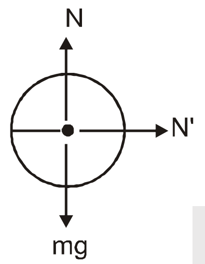
PART - I

Section (A)

A-1. Gravitational, Electromagnetic, Nuclear.

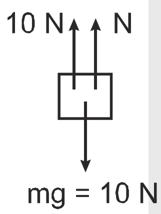
A-2. 20 N **A-3.** No

A-4.



Vertical wall does not exert force on sphere (N' = 0).

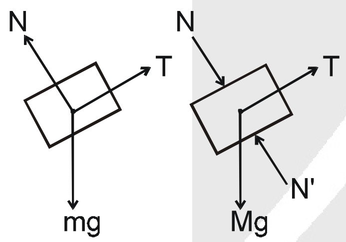
A-5. (i)



(N = 0).

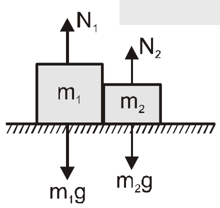
(ii) Gravitational between earth and block.

A-6.

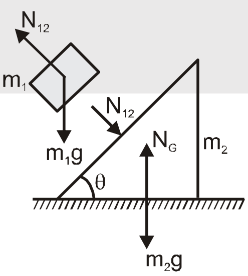


Normal reaction

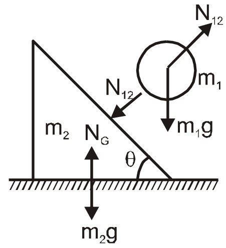
A-7. (a)



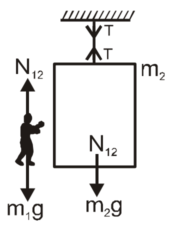
(b)



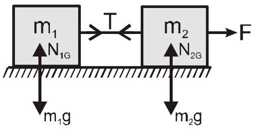
(c)



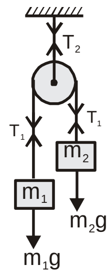
(d)



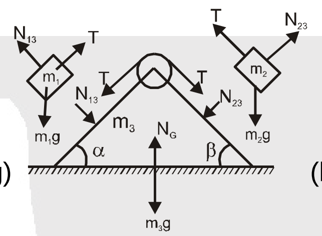
(e)



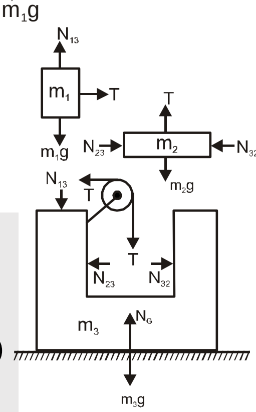
(f)



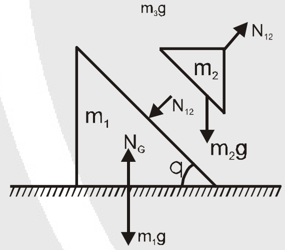
(g)



(h)



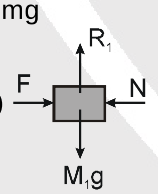
(i)

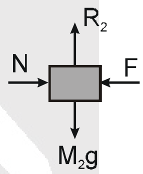


Section (B)

B-1. 2 mg

B-2. (i)



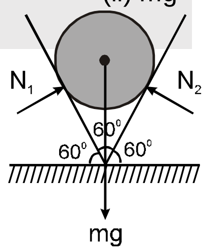


(ii) N = F

(iii) F (iv) m_1g, m_2g.

B-3. (i) zero (ii) mg

B-4. (a)



(b) equal magnitude w .

B-5. $N_A = \dfrac{1000}{\sqrt{3}}$ N, $N_B = \dfrac{500}{\sqrt{3}}$ N

B-6. $N_{45} = \dfrac{50\sqrt{2}}{\sqrt{3}-1} = 96.59$ N; $N_{30} = \dfrac{100}{\sqrt{3}-1} = 136.6$ N

Section (C)

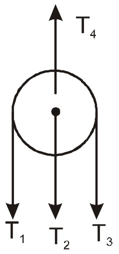
C-1. (a) 10 N, (b) 10 N, (c) 10 N.

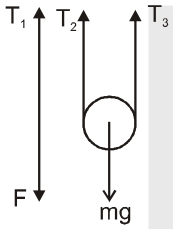
C-2. (a) 10 N (b) 15 N (c) 20 N

C-3. $T_1 = 5N$, $T_2 = 2N$

C-4. (a) $\frac{2g}{3} = 6.7$ m (b) 40 N (c) 80 N

C-5. (a)





(b) $T_1 = T_2 = T_3 = \frac{Mg}{2}$, $T_5 = Mg$ and $T_4 = \frac{3Mg}{2}$

(c) $F = \frac{Mg}{2}$

Section (D)

D-1. $\frac{\cos\theta_2}{\cos\theta_1}$

D-2. 2u

D-3. $V_B = 3\,V_A = 1.8$ m/s in downward direction.

D-4. $a_A = \frac{9g}{25}$, $a_B = \frac{12g}{25}$

D-5. $V_{P_1} = 5$ m/s downward

$V_C = 25$ m/s upward

D-6. $b\,\hat{i} + b\,\hat{j}$

Section (E)

E-1. (a) $|\vec{F}_1| = |\vec{F}_2| = 30\sqrt{2}$ N (b) $W = 30\sqrt{2}$ N

E-2. $|F| = \sqrt{(30)^2 + (108)^2} = 112.08$ N

E-3. $\frac{m_1 g}{2(m_1 + m_2)}$

E-4 (a) 4.8 N, 3.6 N, 2.4N, 1.2 N

(b) F = 6 N (c) 0.2 N

E-5. 6.5 m/s

E-6. (a) 2g/3 (b) 3g/4

E-7. (a) $m_B = 10$ kg (b) $m_B = 10$ kg

Section (F)

F-1. (i) 600 N, (ii) 600 N, (iii) 600 N, (iv), 720 N, (v) 480 N, (vi) 720 N, (vii) 720 N, (viii) 480 N

F-2. (i) 100 N, (ii) 100 N, (iii) 100 N, (iv) 120 N, (v) 80 N, (vi) 120 N, (vii) 120 N, (viii) 80 N.

F-3. **(a) 3 g↓, 0, 0,** **(b) 0, g ↑, g ↓**

Section (G)

G-1. $\frac{15}{4}$ m/s^2, opposite direction.

G-2. $a = \frac{4F}{M+m} - g$.

G-3. 322 N

Section (H)

H-1. F = 0

H-2. $(g + a)\sin\theta$

PART - II

Section (A)

A-1. (C) **A-2.** (B)

Section (B)

B-1. (B) **B-2.** (C)

Section (C)

C-1. (B) **C-2.** (D) **C-3.** (C)
C-4. (C) **C-5.** (C)

Section (D)

D-1. (B) **D-2.** (C) **D-3.** (D)
D-4. (B) **D-5.** (D) **D-6.** (A)

Section (E)

E-1. (D) **E-2.** (C) **E-3.** (B)
E-4. (C) **E-5.** (C) **E-6.** (C)
E-7. (D) **E-8.** (C) **E-9.** (B)
E-10. (D) **E-11.** (C)

Section (F)

F-1. (A) **F-2.** (D) **F-3.** (C)
F-4. (D) **F-5.** (B)

Section (G)

G-1. (B) **G-2.** (A)

Section (H)

H-1. (C) **H-2.** (C) **H-3.** (C)

PART - III

1. (A) q (B) r (C) q (D) r
2. (A) ps,(B) qr , (C) ps, (D) qr
3. (A) r (B) p (C) s (D) q

EXERCISE-2

PART - I

1.	(C)	**2.**	(C)	**3.**	(D)
4.	(A)	**5.**	(A)	**6.**	(B)
7.	(A)	**8.**	(B)	**9.**	(D)
10.	(C)	**11.**	(A)	**12.**	(C)
13.	(B)	**14.**	(C)	**15.**	(D)
16.	(D)				

PART - II

1.	5	**2.**	6	**3.**	3
4.	4	**5.**	1	**6.**	17
7.	60	**8.**	23	**9.**	6
10.	2	**11.**	40		

PART - III

1.	(A) (C)	**2.**	(A)(B)(C)	**3.**	(A)(C)
4.	(A) (B) (D)	**5.**	(B)(C)		
6.	(A)(B)(C)(D)	**7.**	(C) (D)	**8.**	(C)(D)
9.	(A)(B)	**10.**	(B) (D)	**11.**	(B) (D)

PART - IV

1.	(B)	**2.**	(D)	**3.**	(A)
4.	(C)	**5.**	(B)	**6.**	(C)
7.	(A)	**8.**	(B)	**9.**	(C)

EXERCISE - 3

PART - I

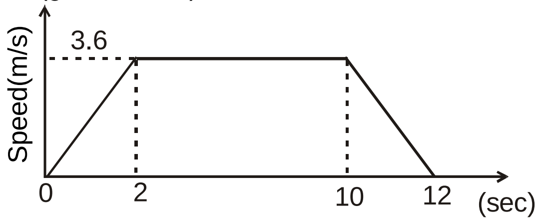
1. (B)

PART - II

1. (4) **2.** (3) **3.** (1) **4.** (3)

High Level Problems (HLP)

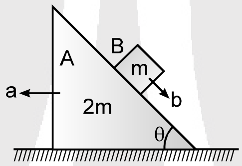
1. A lift is going up. The total mass of the lift and the passengers is 150 kg. The variation in the speed of the lift is given in the graph. ($g = 9.8\ m/s^2$)



(a) What will be the tension in the rope pulling the lift at t equal to
(i) 1 sec (ii) 6 sec and (iii) 11 sec ?
(b) What is the height through which the lift takes the passengers ?
(c) What will be the average velocity and average acceleration during the course of entire motion?

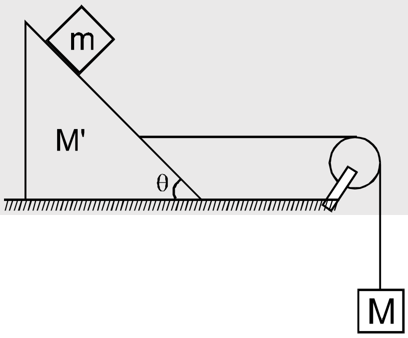
[IIT 1976]

2. The system shown in figure is released from rest calculate the value of acclerations 'a' and 'b'. (Where b is w.r.t. to A)



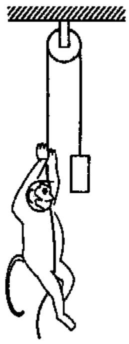
3. A person is standing on a weighing machine placed on the floor of an elevator. The elevator stars going up with some acceleration, moves with uniform velocity for a while and finally decelerates to stop. The maximum and the minimum weights recorded are 80.5 kg and 59.5 kg. Assuming that the magnitudes of the acceleration and the deceleration are the same, find (a) the true weight of the person and (b) the magnitude of the acceleration. Take $g = 10\ m/s^2$.

4. What will be the value M of the hanging block as shown in the figure which will prevent the smaller block from slipping over the triangular block. All the surface are frictionless and the string and the pulley are light.

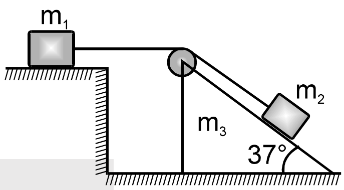


5. A monkey of mass 15 kg is climbing on a rope with one end fixed to the ceiling .If it wishes to go up with an acceleration of $1\ m/s^2$, how much force should it apply to the rope? If the rope is 5m long and the monkey starts from rest, how much time will it take to reach the ceiling ?

6. Figure shown a monkey is climbing on a rope that goes over a smooth light pulley and a block of equal mass hanging on the other end. Show that the monkey and the block move in the same direction with equal acceleration, whatever force the monkey exerts on the rope. If intially both were at rest, their separation will not change as time passes.



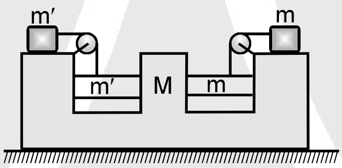
7. In the arrangement shown in Fig, a wedge of mass m_3 = 3.45 kg is placed on a smooth horizontal surface. A small and light pulley is connected on its top edge, as shown. A light, flexible thread passes over the pulley. Two blocks having mass m_1= 1. 3 kg and m_2= 1.5 kg are connected at the ends of the thread. m_1 is on smooth horizontal surface and m_2 rests on inclined surface of the wedge. Base length of wedge is 2 m and inclination is 37^0. m_2 is initially near the top edge of the wedge. If the whole system is released from rest. Calculate:



(i) velocity of wedge when m_2 reaches its bottom

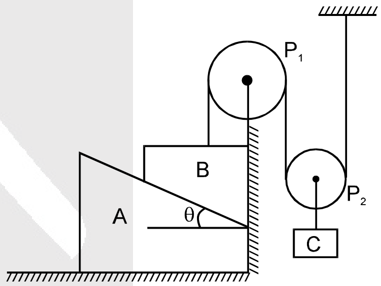
(ii) velocity of m_2 at that instant and tension in the thread during motion of m_2. All the surfaces are smooth. [g = 10 ms^{-2}]

8. Neglecting friction every where, find the acceleration of M. Assume m > m′.

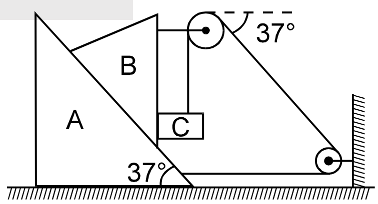


9. In the figure shown P_1 and P_2 are massless pulleys. P_1 is fixed and P_2 can move. Masses of A, B and C are $\frac{9m}{64}$, 2m and m respectively. All contacts are smooth and the string is massless. $\theta = \tan^{-1}\left(\frac{3}{4}\right)$.

Find the acceleration of block C in m/s^2.



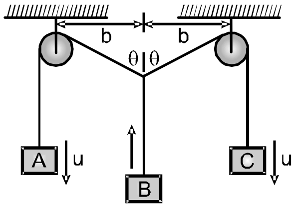
10. A system is shown in figure. All contact surfaces are smooth and string is tight & inextensible. Wedge 'A' moves towards right with speed 10 m/s & velocity of 'B' relative to 'A' is in downward direction along the incline having magnitude 5m/s. Find the horizontal and vertical component of velocity of Block 'C'.



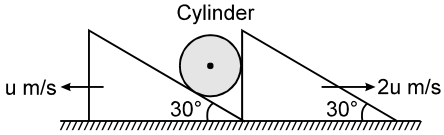
11. An object of mass 2 kg is placed at rest in a frame (S_1) moving with velocity $10\hat{i}+5\hat{j}$ m/s and having acceleration $5\hat{i}+10\hat{j}$ m/s^2. This object is also seen by an observer standing in a frame (S_2) moving with velocity $5\hat{i}+10\hat{j}$ m/s.

(i) Calculate 'Pseudo force' acting on object. Which frame is responsible for this force.
(ii) Calculate net force acting on object with respect to S_2 frame.
(iii) Calculate net force acting on object with respect to S_1 frame.

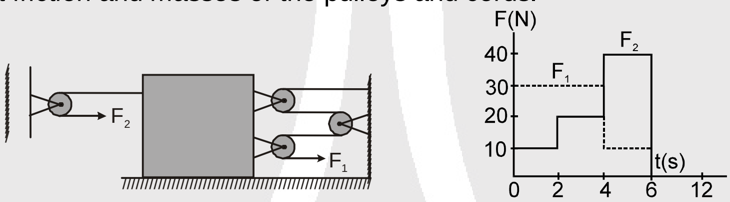
12. In the figure shown the blocks A & C are pulled down with constant velocities u . Find the acceleration of block B.



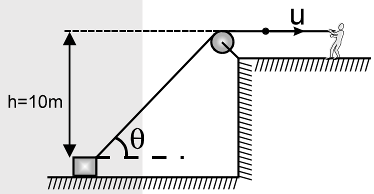
13. System is shown in the figure. Assume that cylinder remains in contact with the two wedges. Find the velocity of cylinder.



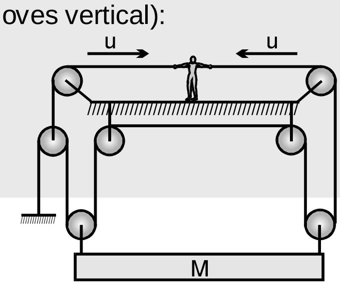
14. The 40 kg block is moving to the right with a speed of 1.5 m/s when it is acted upon by the forces F_1 and F_2. These forces vary in the manner shown in the graph. Find the velocity (in m/s) of the block at t = 12 s. Neglect friction and masses of the pulleys and cords.



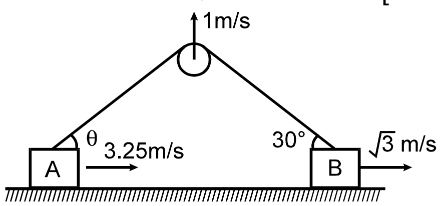
15._ In the figure shown, a person pulls a light string with a constant speed u = 10 m/s. The other end of the string is tied to a very small block which moves on a smooth horizontal surface. The block is initially situated at a distance from the pulley which is very large in comparison to h. Find the angle 'θ' when the block leaves the surface. Take g = 10 m/s^2.



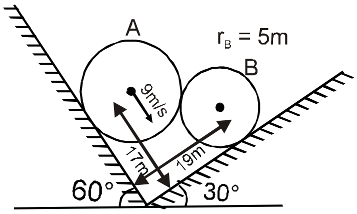
16. System is shown in the figure and man is pulling the rope from both sides with constant speed ' u'. Then find the speed of the block. (M moves vertical):



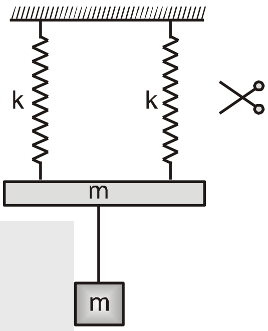
17. In the figure shown, find out the value of θ at this instant [assume string to be tight]



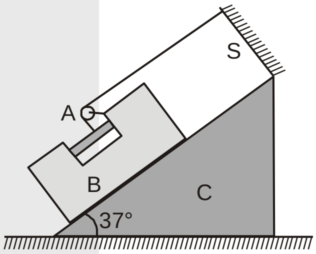
18. System is shown in the figure. Velocity of sphere A is 9 m/s. Find the speed of sphere B.



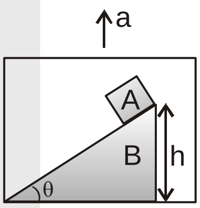
19. System shown in figure is in equilibrium. Find the magnitude of change in tension in the string just before and just after, when one of the spring is cut. Mass of both the blocks is same and equal to m and spring constant of both springs is k. (Neglect any effect of rotation)



20. In the figure shown C is a fixed wedge. A block B is kept on the inclined surface of the wedge C. Another block A is inserted in a slot in the block B as shown in figure. A light inextensible string passes over a light pulley which is fixed to the block B through a light rod. One end of the string is fixed and other end of the string is fixed to A.S is a fixed support on the wedge. All the surfaces are smooth. Masses of A and B are same. Then find the magnitude of acceleration of A. (sin 37º = 3/5)



21. A lift is moving upwards with a constant acceleration a = g. A small block A of mass ' m ' is kept on a wedge B of the same mass ' m '. The height of the vertical face of the wedge is ' h'. A is released from the top most point of the wedge. Find the time (in second) taken by A to reach the bottom of B. All surfaces are smooth and B is also free to move. If h = 4m, θ = 30° and g = 10m/s^2



HLP Answers

1. (a) (i) 1740 N (ii) 1470 N (iii) 1200 N (b) 36 m (c) Average velocity = 3 m/s; Average acceleration = 0

2. $a = \frac{b\cos\theta}{3}$; $b = \frac{3g\sin\theta}{3-\cos^2\theta}$ **3.** 70 kg and 1.5 m/s^2 **4.** $\frac{M'+m}{\cot\theta - 1}$ **5.** 165 N, $\sqrt{10}$ s

7. **(i)** 2 ms^{-1} **(ii)** $\sqrt{13}$ ms^{-1}, 3.9 Newton **8.** $a = \frac{(m-m')g}{2M+3m+3m'}$ **9.** 3m/s^2 upwards

10. Horizontal component of velocity is 14 m/sec and vertical component of velocity is 26 m/sec.

11. (i) F = $-10\hat{i} - 20\hat{j}$ N, Due to acceleration of frame s_1 (ii) $10\hat{i} + 20\hat{j}$ N (iii) zero.

12. $\frac{u^2}{b}\tan^3\theta$ **13.** $\sqrt{7}u$ m/s **14.** 12 m/s. **15.** $\theta = \frac{\pi}{4}$ **16.** $\frac{3u}{4}$

17. $\tan^{-1}\frac{3}{4}$ **18.** 12 m/s **19.** $\frac{mg}{2}$ **20.** $\frac{4\sqrt{2}}{3}$ m/s **21.** 1

SOLUTIONS OF NEWTONS LAWS OF MOTION

EXERCISE-1

PART - I

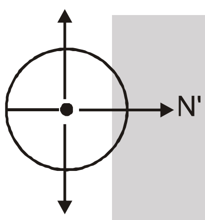
SECTION (A) :

A-1. Gravitational, Electromagnetic, Nuclear.

A-2. Newton's III[rd] Law

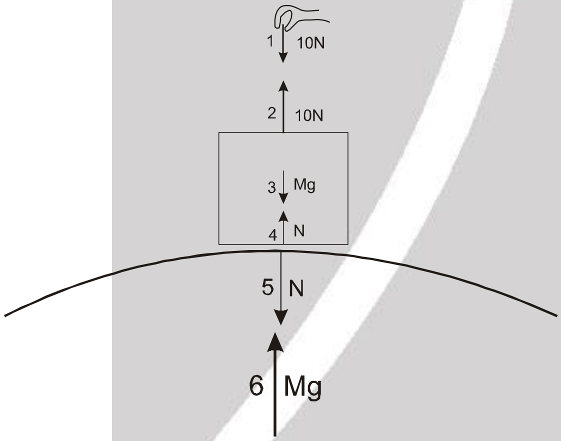
A-3. Newton's II[nd] Law

A-4.



Vertical wall does not exert force on sphere (N' = 0).

A-5.



For block

mg = 10 +N [Equilibrium]

$\Rightarrow$ $1\times 10 = 10 + N$

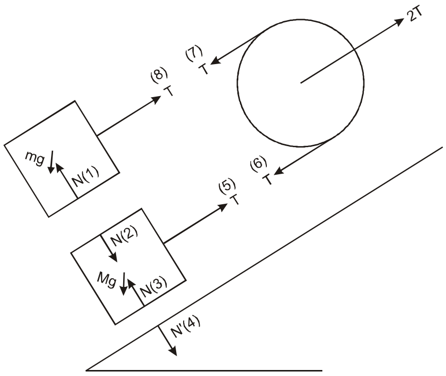
$\Rightarrow$ N = 0

(1) and (2) are action reaction pair

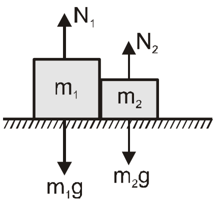
(3) and (6) are action reaction pair

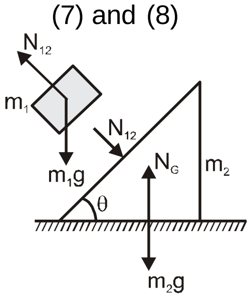
(4) and (5) are action reaction pair

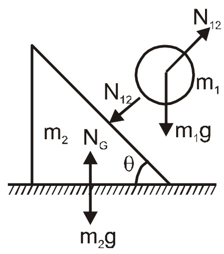
A-6.

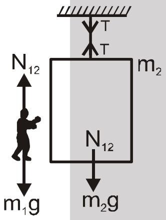


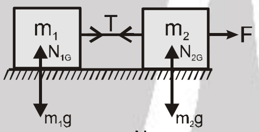
Action reaction pairs (1) and (2)
(3) and (4)
(5) and (6)
(7) and (8)

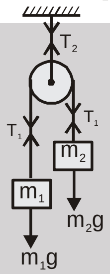
A-7. (a)


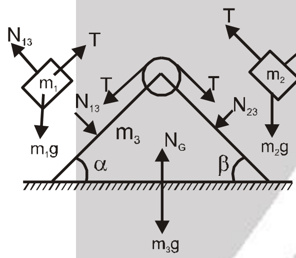
(b)


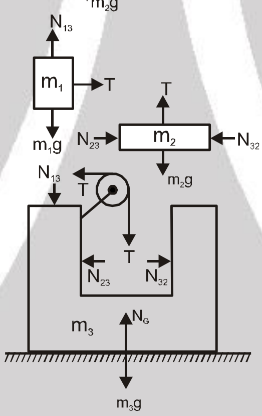
(c)


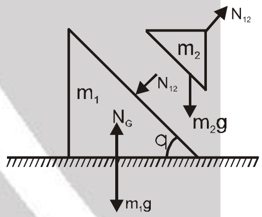
(d)


(e)


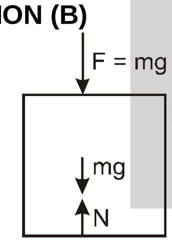
(f)


(g)


(h)


(i)


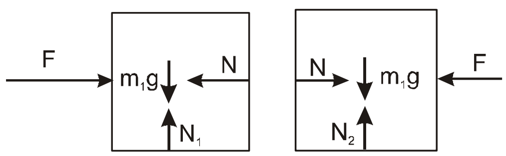
SECTION (B)

B-1.


N = F + mg [equilibrium]

$\Rightarrow$ N = mg + mg

$\Rightarrow$ N = 2mg

B-2.


It is obvious that both blocks will have same acceleration. If we take both block as one system then.

$F - F = (m_1 + m_2)\, a$ $\begin{bmatrix} \text{Newton's second law} \\ \text{in horizontal direction} \end{bmatrix}$

$\Rightarrow$ a = 0

Now take m_1 as a system

$$\begin{bmatrix}\text{Newton's second law}\\ \text{in horizontal direction}\end{bmatrix}$$

$F - N = m_1 a$

$\Rightarrow$ $F - N = 0$

$\Rightarrow$ $F = N$

$m_1g - N_1 = 0$ [Equilibrium in vertical direction]

Now take m_2 as system

$N - F = m_2 a$ $\begin{bmatrix}\text{Newton's second law}\\ \text{in horizontal direction}\end{bmatrix}$

$\Rightarrow$ $N - F = 0$

$N = F$

$m_2g - N_2 = 0$ [Equilibrium in vertical direction]

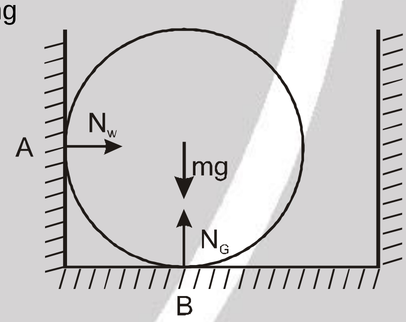
$\Rightarrow$ $N_2 = m_2g$

B-3. The sphere is in contact at two surfaces one at wall and one at ground.
So one Normal reaction can be exerted at A and another at B.

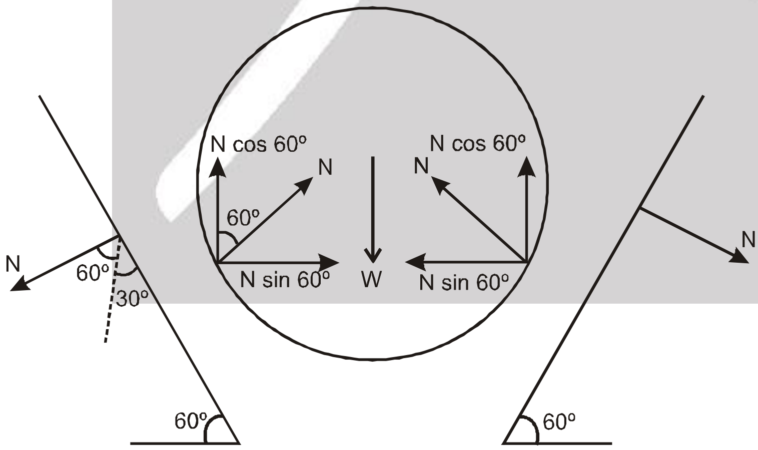
$N_w = 0$ [Equilibrium in horizontal direction]

$N_G - mg = 0$ [Equilibrium in vertical direction]

$\Rightarrow$ $N_G = mg$



B-4.



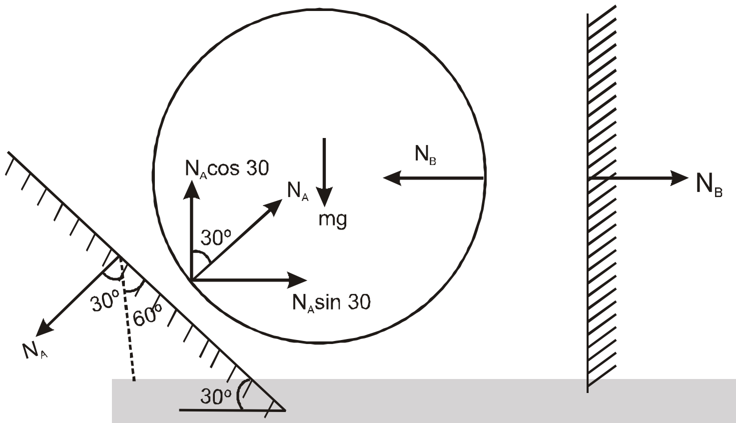
Due to symmetry normal reactions due to left and right wall are same in magnitude

$W - N\cos 60 - N\cos 60 = 0$ [Equilibrium in vertical direction]

$\Rightarrow$ $W - \frac{N}{2} - \frac{N}{2} = 0$

$\Rightarrow$ $N = W$

B-5.



$mg - N_A \cos 30 = 0$ [Equilibrium in vertical direction]

$\Rightarrow \quad N_A = \dfrac{mg}{\cos 30}$

$\Rightarrow \quad N_A = \dfrac{1000}{\sqrt{3}}$ N

$N_B - N_A \sin 30 = 0$ [Equilibrium in horizontal]

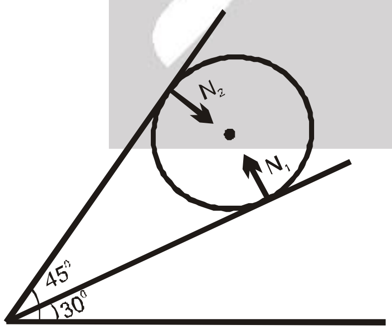
$\Rightarrow \quad N_B = N_A \sin 30$

$\Rightarrow \quad N_B = \dfrac{1000}{\sqrt{3}} \, \dfrac{1}{2}$

$\Rightarrow \quad N_B = \dfrac{500}{\sqrt{3}}$ N

B-6. $N_1 \cos 30° = 50 + \dfrac{N_2}{\sqrt{2}}$

$N_1 \dfrac{\sqrt{3}}{2} - \dfrac{N_2}{\sqrt{2}} = 50$ .... (1)



$N_1 \sin 30° = \dfrac{N_2}{\sqrt{2}}$

$N_1 = \sqrt{2}\, N_2$ ....(2)

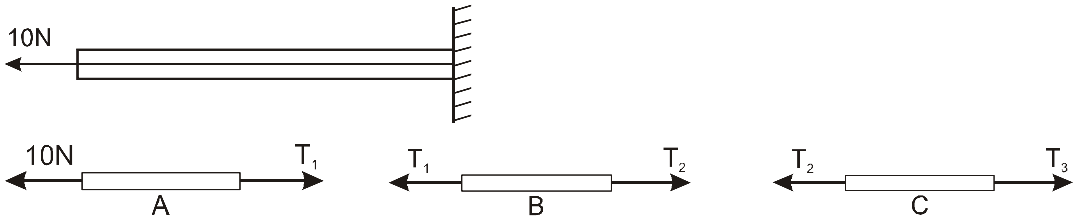
Solving equation (1) & (2)

$N_1 = 136.6$ N

$N_2 = 96.59$ N

SECTION (C) :

C-1.

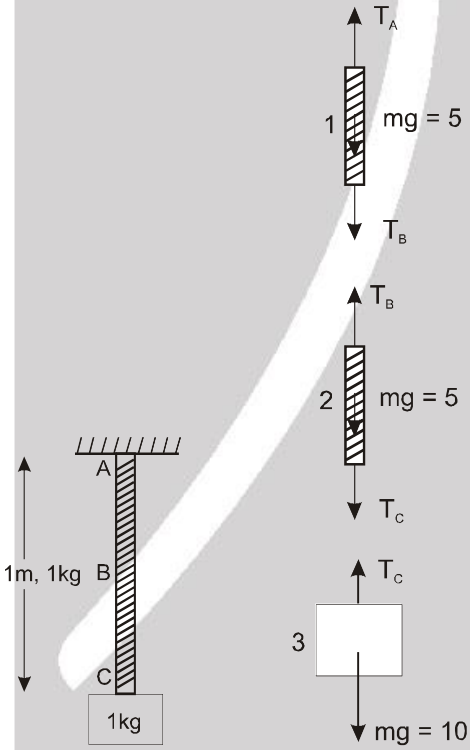


$T_1 - 10 = 0$ [Equilibrium of A]

$T_1 = 10$ N

Similarly T_2 and T_3 are also 10 N

C-2. (a) 10 N (b) 15 N (c) 20 N



$T_C - 10 = 0$ [Equilibrium of block]

$\Rightarrow$ $T_C = 10$ N

$T_B - T_C - 5 = 0$ [Equilibrium of 2]

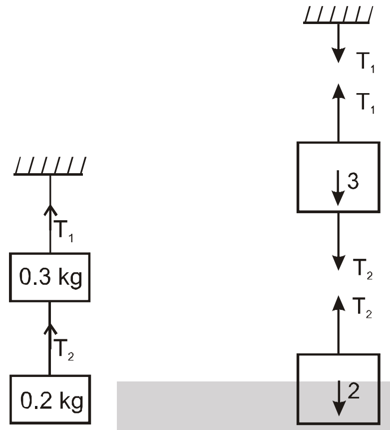
$\Rightarrow$ $T_B - 10 - 5 = 0$

$\Rightarrow$ $T_B = 15$ N

$T_A - T_B - 5 = 0$ [Equilibrium of 1]

$\Rightarrow$ $T_A = 20$ N

C-3.



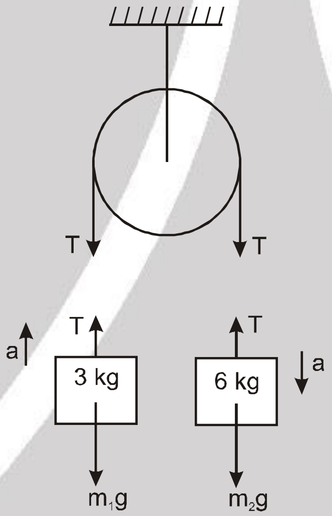
$T_2 - 2 = 0$ [Equilibrium of lower block]

⇒ $T_2 = 2N$

$T_1 - T_2 - 3 = 0$ [Equilibrium of upper block]

⇒ $T_1 = 5$ N

C-4. For finding distance travelled we need to know the acceleration and initial velocity of block.



$m_2g - T = m_2a$ [Newton's second law for m_2]

$T - m_1g = m_1a$ [Newton's second law for m_1]

$m_2g - m_1g = (m_2 + m_1)a$ [adding both the equation]

⇒ $a = \frac{(m_2 - m_1)g}{m_2 + m_1}$

$a = \frac{6-3}{6+3} \times g$

$a = \frac{g}{3} = \frac{10}{3}$ m/s^2

$s = ut + 1/2\ at^2$

$= 0 \times 2 + 1/2 \times 10/3 \times 2^2$

$S = \frac{20}{3}$ m

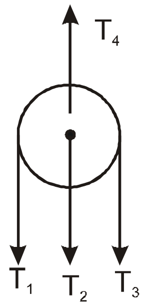
$T - m_1g = m,\ a$

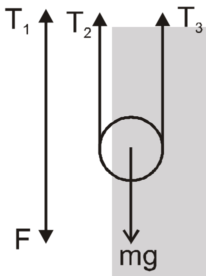
$T = m_1\left(g + \frac{g}{3}\right) = 3 \times 40/3$ $T = 40$ N

Force exerted by clamp on pulley is 2T

⇒ $2 \times 40 = 80$ N

C-5.





$T_1 = F$ [Equilibrium of string]

$T_3 = T_1$ [String is massless and pulley is friction less so tension must be same on both sides of string]

$\Rightarrow \quad T_3 = F$

Similarly $T_2 = F$

$T_5 = T_2 + T_3$ [Equilibrium of lower pulley]

$\Rightarrow T_5 = 2F$

$T_5 = mg$ [Equilibrium of block]

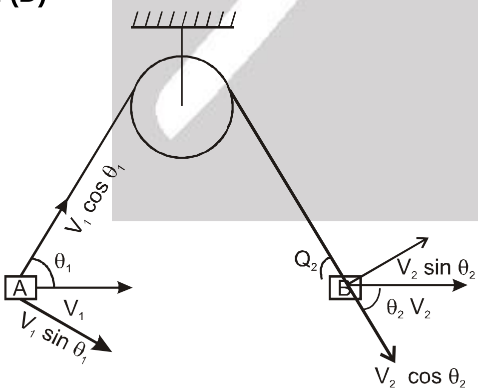
$F_1 = T_2 = T_3 = \frac{Mg}{2}$ $\quad T_4 = T_1 + T_2 + T_3$

[Equilibrium of upper pulley]

$\Rightarrow T_4 = \frac{3 \ Mg}{2}$

Section (D)

D-1.



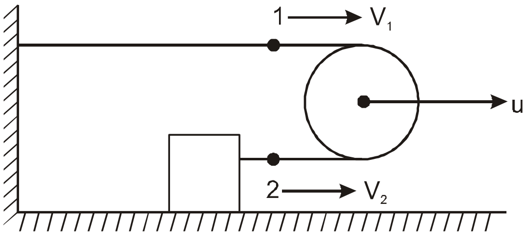
Since string is inextensible length of string can't change

$\therefore$ Rate of decreases of length of left string = rate of increase of length of right string

$\Rightarrow \quad V_1 \cos\theta_1 = V_2 \cos\theta_2$

$\Rightarrow \quad \frac{V_1}{V_2} = \frac{\cos\theta_2}{\cos\theta_1}$

D-2.



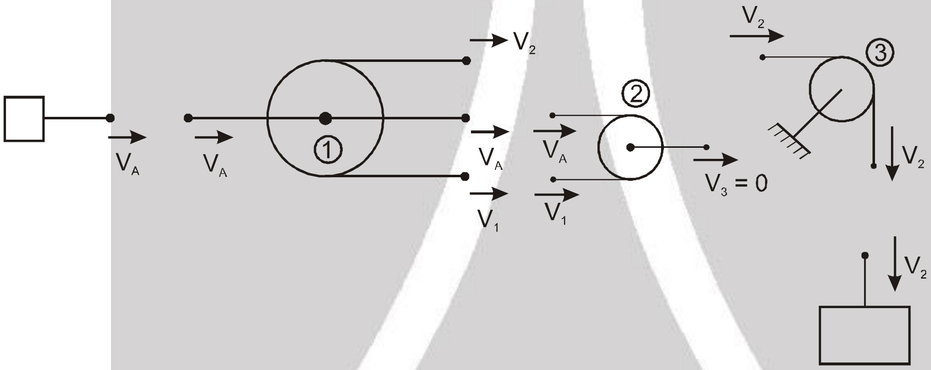
Velocity of point 1 is V_1 which is 0 because string is fixed.
Velocity of point 2 is V_2

$$\frac{V_1 + V_2}{2} = u$$

$$\frac{0 + V_2}{2} = u$$

$$V_2 = 2u$$

D-3.



$V_A = \frac{V_1 + V_2}{2}$ [Pulley 1]

$V_1 + V_2 = 2V_A$ I

$\frac{V_A + V_1}{2} = V_3$ [Pulley 2]

$V_A + V_1 = 0$ II

$\Rightarrow$ $V_1 = -0.6$

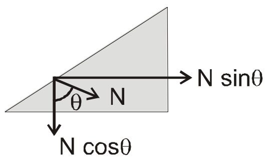
$-0.6 + V_2 = 2V_A$

$V_2 = 2 \times 0.6 + 0.6$

$V_2 = 1.8$ m/s

$V_B = V_2 = 1.8$ m/s

D-4. $\tan 37^o = \frac{a_A}{a_B}$ (wedge constrained relation)

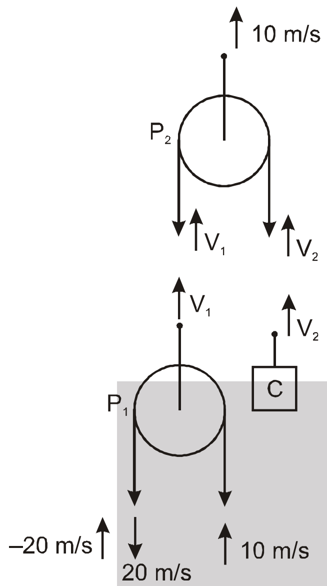


$N \sin 37° = ma_B$ ....(i)

For Rod $\rightarrow mg - N \cos 37° = ma_A$ ....(ii)

From equation (1) & (2) $a_A = \frac{9g}{25}$, $a_B = \frac{12g}{25}$

D-5.



$V_1 = \frac{10-20}{2}$ [constrained relation of P_1]

$V_1 = -5$ m/s

$10 = \frac{-5+V_2}{2}$

$V_2 = 25$ m/s ↑

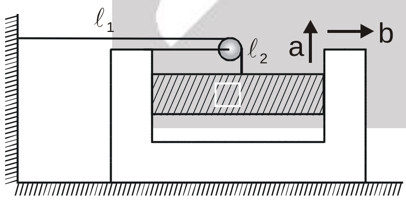
$V_C = V_2 = 25$ m/s upwards

$V_{P_1} = V_1 = -5$ m/s

$\Rightarrow V_P = 5$ m/s downward

[because we have assumed upward direction as +ve for V_1]

D-6.



$\ell_1 + \ell_2 = C$

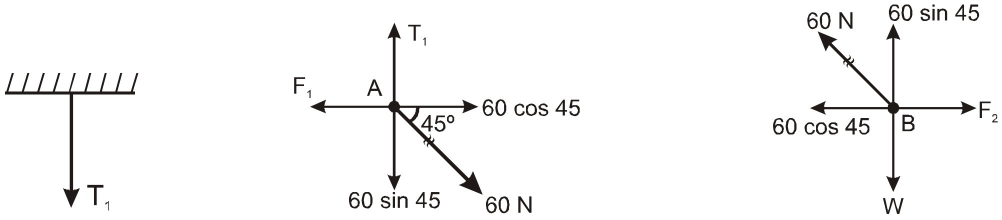
$\ell_1'' + \ell_2 = 0$

$b - a = 0$ $a = b$

Acceleration of A $\mathbf{b}\hat{i} + \mathbf{b}\hat{j}$

SECTION (E)

E-1.



Since point A is massless net force on it must be zero other wise it will have ∞ acceleration.

⇒ $F_1 - 60 \cos 45 = 0$

⇒ $F_1 = 30\sqrt{2}$ N

$F_2 - 60 \cos 45 = 0$

$F_2 = 30\sqrt{2}$ N

$W - 60 \sin 45 = 0$

$W = 30\sqrt{2}$ N

E-2. $\vec{F} = m\vec{a}$

$\vec{a} = a_x\hat{i} + a_y\hat{j}$

$= \frac{d^2x}{dt^2} + \frac{d^2y}{dt^2}\ \hat{i}$

$= (10)\ \hat{i} + (18\ t)\ \hat{j}$

at t = 2 sec t = 2 sec

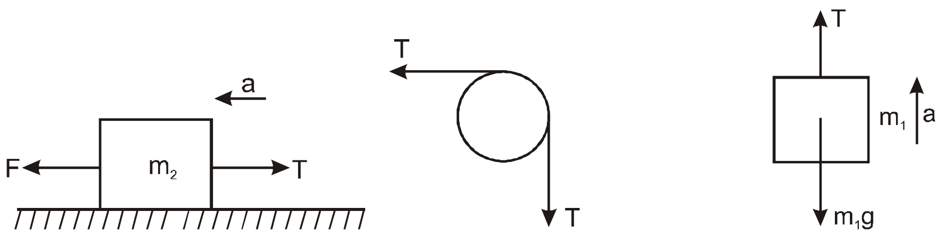
$\vec{a} = 10\ \hat{i} + 36\hat{j}$

$\vec{F} = 3\ (10\hat{i} + 36\hat{j})$

$= 30\hat{i} + 108\hat{j}$

$|\vec{F}| = \sqrt{30^2 + 108^2} = 112.08$ N

E-3.



It is obevious that acceleration of both the blocks is same in magnitude.

$F - T = m_2a$ [Newtons second law for m_2]

$T - m_1g = m_1a$ [Newtons second law for m_1]

After adding the above equations.

$F - m_1g = (m_2 + m_1)a$

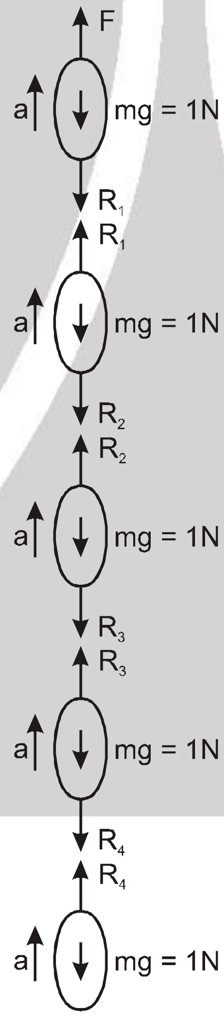
$\frac{m_1g}{2} - m_1g = (m_2 + m_1)a$

$\Rightarrow \quad a = -\frac{m_1g}{2(m_1+m_2)}$

The value of a is –ve it means

$a = \frac{m_1g}{2(m_1+m_2)}$ in the direction opposite to assumed direction

E-4.



$R_4 - mg = ma$

$R_4 - 1 = 0.1 \times 2$

$R_4 = 1.2$ N

$R_3 - mg - R_4 = ma$

$R_3 - 1 - 1.2 = 0.1 \times 2$

$\Rightarrow \quad R_3 = 2.4$ N

Similarly

$R_2 = 3.6$ N

$R_1 = 4.8$ N

$F = 6$ N

$F_{net} = ma$

$= 0.1 \times 2$

$= 0.2$ N

E-5. $\int dp = p_f - p_i = \int F \, dt$ = Area under the curve.

$p_i = 0$

Net Area 16 – 2 – 1 = 13 N-s

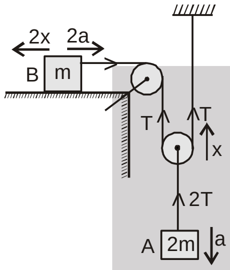
$V_f = 13/2 = 6.5 \, i$ m/s

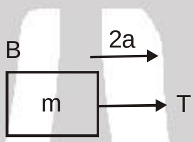
[As momentum is positive, particle is moving along positive x axis.]

E-6. (a) When the block m is pulled 2x towards left the pully rises vertically up by x amount.

$\therefore \; a_B = 2a_A$

F.B.D. of blocks

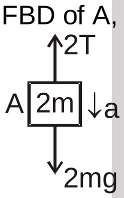




$T = m2a$ ……(1)

F.B.D.

FBD of A,



$2mg - 2T = 2ma$

$mg - T = ma$ …….(2)

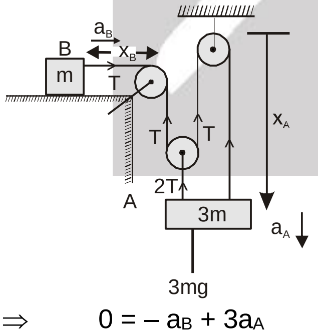
(1) + (2) $\Rightarrow$ $mg = 3ma$

$a = g/3$

$\therefore \quad a_B = 2g/3$

(b) $\ell = x_B + 3x_A$

$\Rightarrow \quad 0 = \frac{d^2x_B}{dt^2} + 3\frac{d^2x_A}{dt^2}$



$\Rightarrow \quad 0 = -a_B + 3a_A$

$\Rightarrow \quad a_B = 3a_A$ ……..(1)

For B,

$T = ma_B$ ……..(2)

For A,

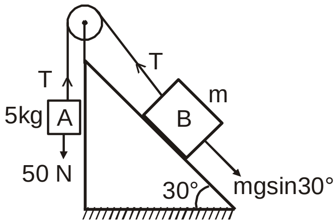
$3mg - 3T = 3ma_A$ ……..(3)

$mg - T = ma_A$

By (1), (2) & (3)

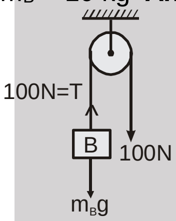
$\therefore \quad a_B = 3g/4$ **Ans.**

E-7. (a)
For Block (A)



T = 50 N ......(1)
For Block (B)
T = mgsin30° ......(2)
∴ $50 = m_B \times 10 \times 1/2$
⇒ $m_B = 10$ kg **Ans.**

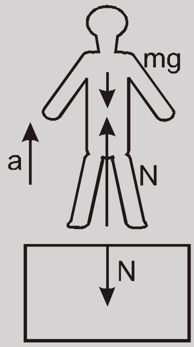
(b)



$T - m_Bg = 0$
⇒ $100 = m_Bg$
∴ $m_B = 10$ kg **Ans.**

SECTION (F)

F-1. Reading of weighing machine is equal to the normal reaction Normal reaction is not affected by velocity of lift, it is only affected by acceleration of lift.
For I, II and III a = 0



N – mg = 0 [Equilibrium of man]
N = mg = 600 N
For IV, VI and VII IV, $a = +2\ m/s^2$
N – mg = ma [Newtons II law]
N = 60 × 2 + 60 × 10 = 720 N
For V and VIII $a = -2\ m/s^2$
N – mg = ma [Newtons II law]
N = 60 × (–2) + 60 × 10 = 480 N

F-2. Reading of spring balance is equal to the tension in spring balance which doesn`t depend on velocity of lift but depend on acceleration.

For I, II and III a = 0 a = 0

$T - 100 = 0$ [Equilibrium]

$T = 100$ N

For IV, VI and VII

$T - 100 = ma$ [Newton's II law]

$T - 100 = 10 \times 2$

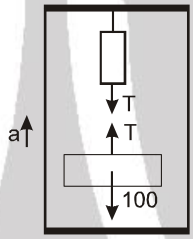
$T = 120$ N

For V and VII

$T - 100 = ma$ [Newton`s II law]

$T - 100 = 10(-2)$

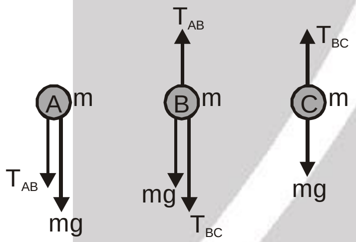
$T = 80$ N



F-3. Initially

$T_{AB} = 2mg$, $T_{BC} = mg$

(a)



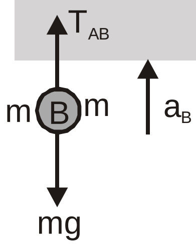
For A $2mg + mg = ma_A$ $\Rightarrow$ $a_A = 3g$

For B $T_{AB} - mg - T_{BC} = ma_B$

$\Rightarrow$ $2mg - mg - mg = ma_B$ $\Rightarrow$ $ma_B = a_B = 0$

$T_{BC} - mg = ma_c$ $\Rightarrow$ $a_c = 0.$

(b)



$T_{AB} = 2mg$

$T_{AB} - mg = ma_B$

$2mg - mg = ma_B$

$\Rightarrow$ $a_B = g\ (\uparrow)$

$a_A = 0$ & $a_C = g(\downarrow)$.

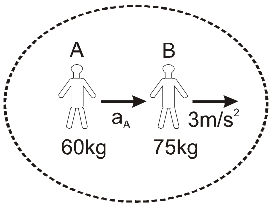
SECTION (G)

G-1. If we take both A and B as a system then there is no external force on system.

$\Rightarrow$ $m_A a_A + m_B a_B = 0$ [Newton’s II law for system]

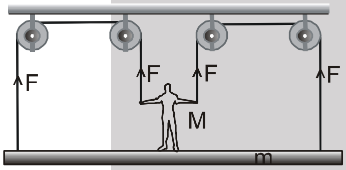
$60\ a_A + 75 \times 3 = 0$

$a_A = \frac{-15}{4}\ m/s^2$



–ve sign means that acceleration is in direction opposite to the assumed direction.

G-2.



$4F - (M + m)g = (M + m)a$

$a = \frac{4F - (M+m)g}{M+m} = \frac{4F}{M+m} - g$

G-3. $T_D = W_{A_{app}} + W_{B_{app}} + W_{C_{app}}$

$T_D = W_{A_{\text{आभासी}}} + W_{B_{\text{आभासी}}} + W_{C_{\text{आभासी}}}$

$= 10(10 - 2) + (15 \times 10) + 8\ (10 + 1.5)$

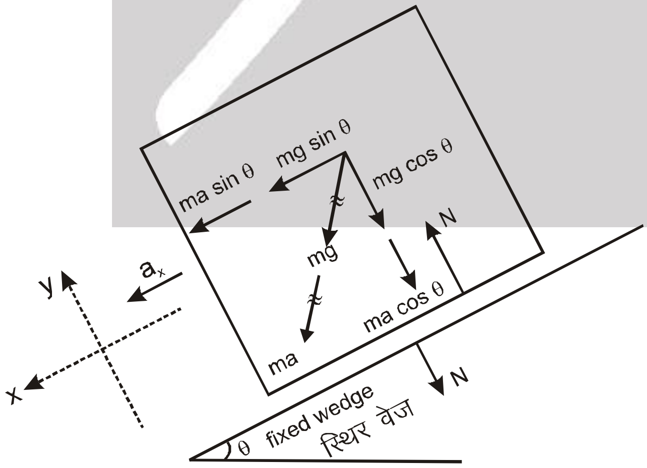
$= 322$ N **Ans.**

SECTION (H)

H-1. Pseudo force depends on mass of object and acceleration of observer (frame) which is zero in this problem.

$\Rightarrow$ Pseudo force is zero.

H-2.



F.B.D. in frame of lift

It is obevious that block can accelerate only in x direction. ma is Pseudo force.

$\Rightarrow$ $mg \sin\theta + ma \sin\theta = ma_x$ [Newton`s II law for block in x direction]

$\Rightarrow$ $a_x = (g + a) \sin\theta$

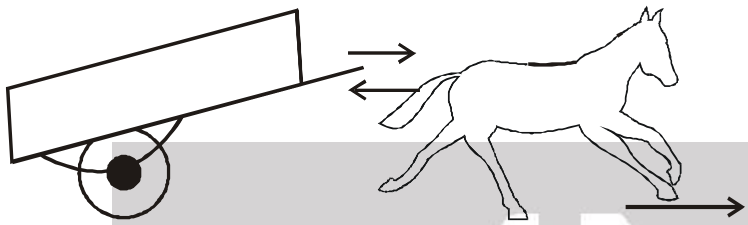
PART - II

SECTION (A)

A-1. Force exerted by string is always along the string and of pull type.

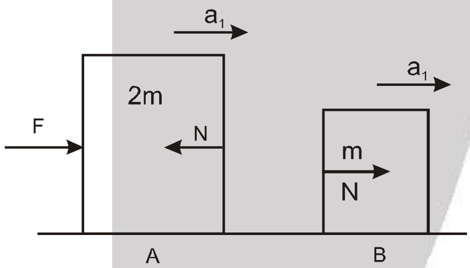
When there is a contact between a point and a surface the normal reaction is perpendicular to the surface and of push type.

A-2. The ground on the horse



SECTION (B)

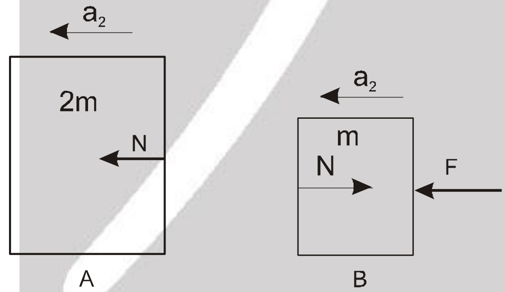
B-1.



$F - N = 2\ ma$, [Newton`s II law for block A] $N = ma_1$

[Newton`s II law for block B]

$\Rightarrow$ $N = F/3$

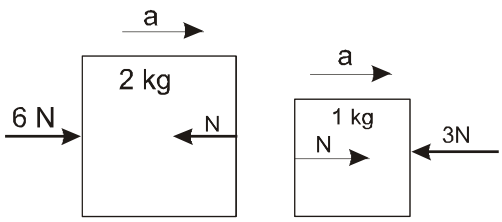


$N = 2\ ma_2$ [Newtons II law for block A]

$F - N = m_2a$ [Newtons II law for block B]

$\Rightarrow$ $N = 2F/3$ so the ratio is 1 : 2

B-2.



Both blocks are constrained to move with same acceleration.

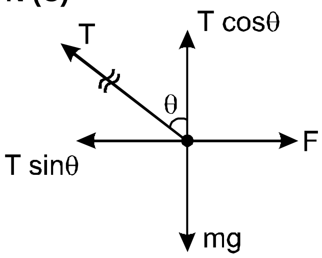
$6 - N = 2a$ [Newtons II law for 2 kg block]

$N - 3 = 1a$ [Newtons II law for 1 kg block]

$\Rightarrow$ N = 4 Newton

SECTION (C)

C-1.

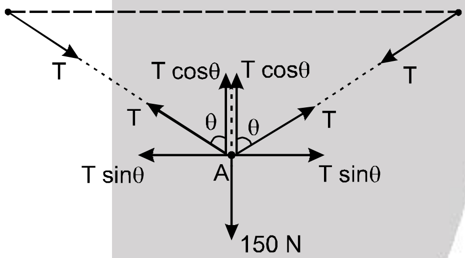


Point A is mass less so net force on it most be zero otherwise it will have ∞ acceleration.

$\Rightarrow$ $F - T\sin\theta = 0$ [Equilibrium of A in horizontal direction]

$\Rightarrow$ $T = \dfrac{F}{\sin\theta}$

C-2.



$T\cos\theta + T\cos\theta - 150 = 0$ [Equilibrium of point A]

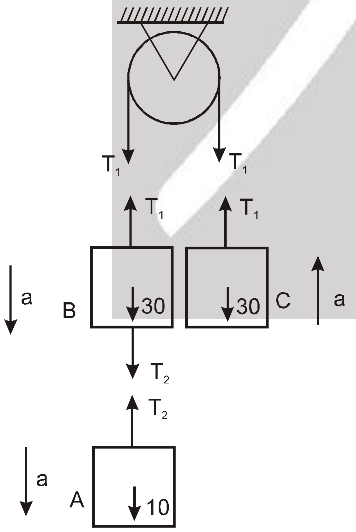
$2T\cos\theta = 150$

$T = \dfrac{75}{\cos\theta}$

When string become straight θ becomes 90º

$\Rightarrow$ $T = \infty$

C-3.



$10 - T_2 = 1\,a$ [Newton's II law for A]

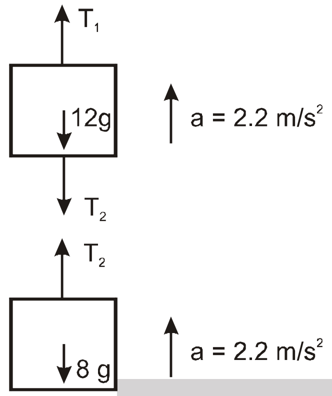
$T_2 + 30 - T_1 = 3\,a$ [Newton's II law for B]

$T_1 - 30 = 3a$ [Newton's II law for C]

$\Rightarrow$ $a = g/7$

$\Rightarrow$ $T_2 = 6g/7$

C-4.



$T_2 - 8g = 8a$ [Newton's II law for 8 kg block]

$\Rightarrow$ $T_2 = 8 \times 2.2 + 8 \times 9.8$

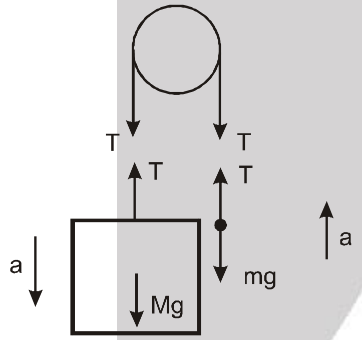
$= 96$ N

$T_1 - 12\ g - T_2 = 12\ a$ [Newton's II law for 12 kg block]

$\Rightarrow$ $T_1 = 12 \times 2.2 + 12 \times 9.8 + 96$

$T_1 = 240$ N

C-5.



$Mg - T = Ma$ [Newton's II law for M]

$T - mg = ma$ [Newton's II law for m]

$\Rightarrow$ T =

If m << M than $m + M \approx M$

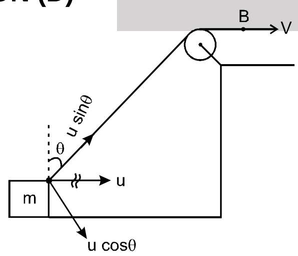
$\Rightarrow$ $T = \dfrac{2\ m\ Mg}{m + M}$

$\Rightarrow$ T = 2 mg

Total downward force on pulley is 2T = 4 mg.

SECTION (D)

D-1.

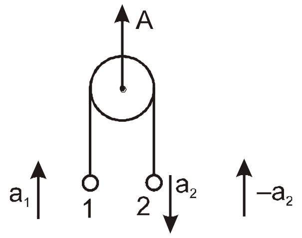


The length of string AB is constant.

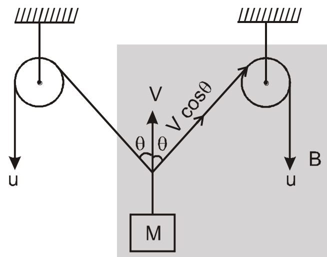
$\Rightarrow$ Speed A and B along the string are same $u \sin\theta = V$

$u \sin\theta = V$ $u = \dfrac{V}{\sin\ \theta}$

D-2. $A = \frac{a_1 - a_2}{2}$



D-3.



By symmetry we can conclude that block will move only in vertical direction.
Length of string AB remains constant
∴ Velocity of point A and B along the string is same.

$V \cos\theta = u \quad \Rightarrow \quad V = \frac{u}{\cos\theta}$

D-4. Let $AB = \ell$, $B = (x , y)$

$\vec{v}_B = v_x\hat{i} + v_y\hat{j}$

$\vec{v}_B = \sqrt{3}\ \hat{i} + v_y\hat{j} \quad \rightarrow \quad (i)$

$x^2 + y^2 = \ell^2$

$2x\, v_x + 2y\, v_y = 0 \quad \Rightarrow \quad \sqrt{3} + \frac{y}{x} v_y = 0$

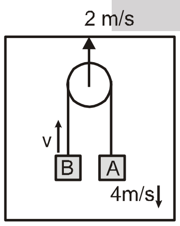
$\Rightarrow \quad \sqrt{3} + (\tan 60°)\, v_y = 0 \quad \Rightarrow \quad v_y = -1$

Hence from (i)

$\vec{v}_B = \sqrt{3}\ \hat{i} - \hat{j}$

Hence $v_B = 2$ m/s

D-5.

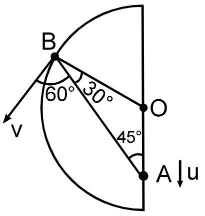


V = (velcoity of B w.r.t ground)

$\frac{V - 4}{2} = 2 \qquad V = 8$ m/s (velcoity of B w.r.t ground)

$V' = 6$ m/s (velcoity of B w.r.t lift)

D-6. u cos 45° = v cos 60°



or v = $\sqrt{2}$ u

SECTION (E)

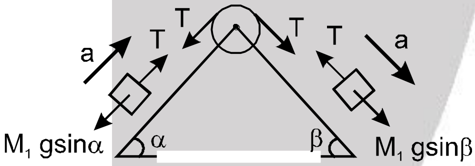
E-1. $\vec{F} = m\vec{a}$

$\vec{a} = \frac{d\vec{v}}{dt}$

E-2. $\vec{F} = m\vec{a}$

E-3. In free fall gravitation force acts.

E-4.



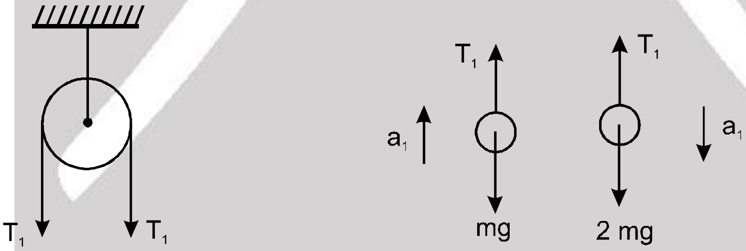
M_2 g sinα – T = M_2a [Newton's II law for M_2]

T – M_1g sinβ = M_1a [Newton's II law for M_1]

By adding both equations

$$a = \left[\frac{M_2 \sin\alpha - M_1 \sin\beta}{M_1 + M_2}\right] g$$

E-5. Case 1

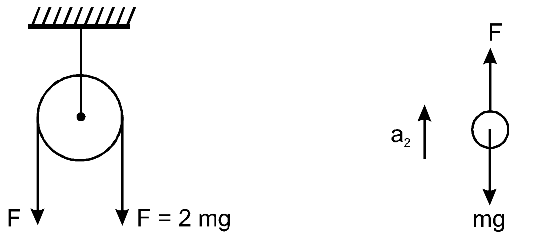


T_1 – mg = ma_1 [Newton's II law for m]

2 mg – T_1 = 2 ma_1 [Newton's II law for 2m]

⇒ a_1 = g/3

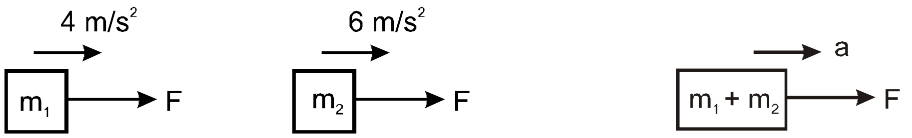
Case 2



F – mg = ma_2 [Newton's II law for m]

⇒ 2 mg – mg = ma_2 ⇒ a_2 = g ⇒ $a_2 > a_1$

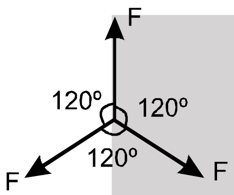
E-6.



$F = m_1 4$ [Newton's II law for m_1]
$F = m_2 6$ [Newton's II law for m_2]
$F = (m_1 + m_2)a$ [Newton's II law for $(m_1 + m_2)$]

$\Rightarrow \quad F = \left[\frac{F}{4} + \frac{F}{6}\right] a \quad \Rightarrow \quad 1 = \left[\frac{1}{4} + \frac{1}{6}\right] a \quad \Rightarrow \quad a = 2.4 \text{ m/s}^2.$

E-7.



Due to symmetry we can say net force on body M is 0.
$\therefore$ Acceleration is 0

E-8. $mg - \frac{3}{4}mg = ma$ [Newton's II law for man]

$\Rightarrow \quad a = g/4$

E-9. $\vec{F} = 6 \;\hat{i} \; -8 \;\hat{j} + 10 \;\hat{k}$

$\vec{F} = m\vec{a}$

$|\vec{F}| = m \; |\vec{a}|$

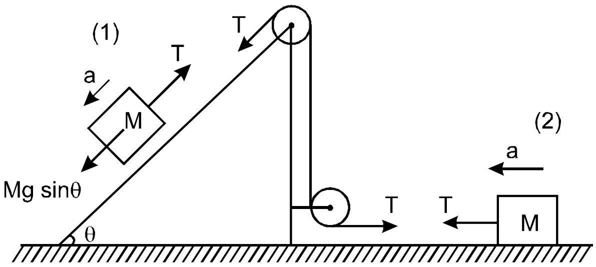
$\sqrt{6^2 + 8^2 + 10^2} = m \; 1 \qquad m = 10\sqrt{2}$ kg.

E-10. $\upsilon^2 = \upsilon^2 + 2 \;\; as \qquad 0^2 = 1^2 + 2 \;\; \frac{F}{m} \;\; x$

$x = \frac{-m}{2F} \qquad \upsilon^2 = \upsilon^2 + 2 \;\; as$

$O^2 = 3^2 + \frac{2F^1}{m} \;\; x \qquad 0 = 9 + \frac{2F^1}{m} \left(\frac{- \;\; m}{2 \;\; F}\right) \Rightarrow \qquad F^1 = 9F$

E-11.

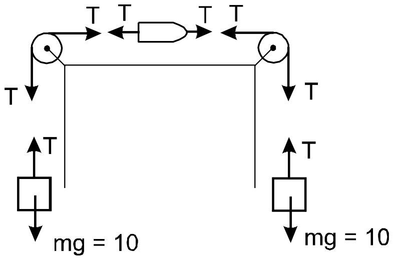


$Mg \sin\theta \; - T = Ma$ [Newton's II law for block 1]
$T = Ma$ [Newton's II law for block 2]
By dividing both equations

$2 \; T = Mg \sin\theta \qquad T = \frac{Mg \;\; \sin\theta}{2}$

SECTION (F)

F-1.



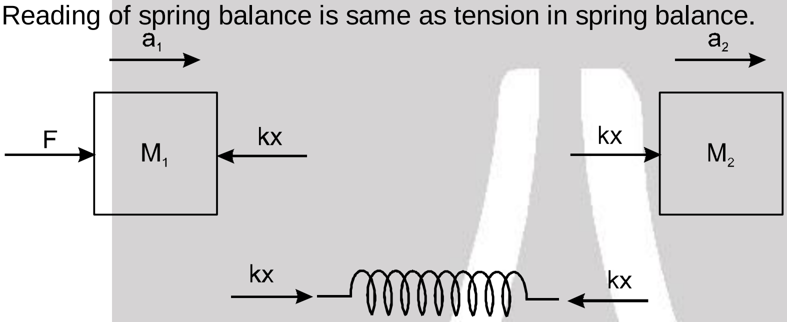
$T - mg = 0$ [Equilibrium of block]

$T - 10 = 0$

$T = 10$

Reading of spring balance is same as tension in spring balance.

F-2.



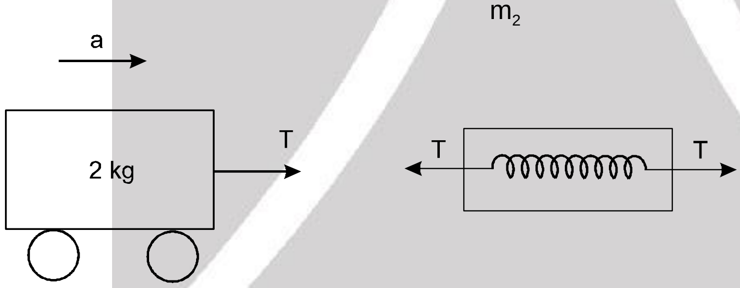
$F - kx = m_1 a_1$ [Newton's II law for M_1]

$kx = m_2 a_2$ [Newton's II law for M_2]

By adding both equations.

$F = m_1 a_1 + m_2 a_2 \Rightarrow a_2 = \dfrac{F - m_1 a_1}{m_2}$

F-3.



Reading of spring balance is same as tension in the balance.

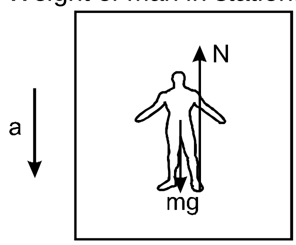
$\Rightarrow \quad T = 10\ g = 98\ N$

$T = 2\ a$

[Newton's II law for 2 kg block]

$\Rightarrow \quad a = 49\ m/s^2$

F-4. Weight of man in stationary lift is mg.



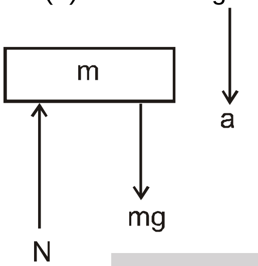
$mg - n = ma$ [Newton's II law for man]

$\Rightarrow \quad N = m(g - a)$

Weight of man in moving lift is equal to N.

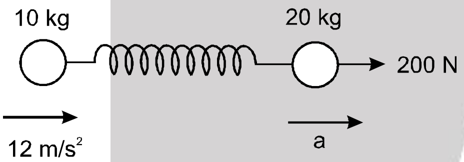
$\Rightarrow \dfrac{m\ g}{m\ (g-a)} = \dfrac{3}{2} \quad \Rightarrow \quad a = \dfrac{g}{3}$

F-5. $N = m(g - a)$, $N < mg$ if a (↓)
$N = m(g - a)$ $N < mg$
and $N > mg$ if a (↑)
Reading of spring balance is less than m
if a (↓) and reading of spring balance is



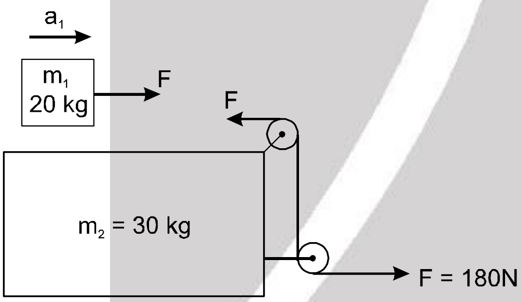
greater than m if a (↑)

SECTION (G)
G-1.



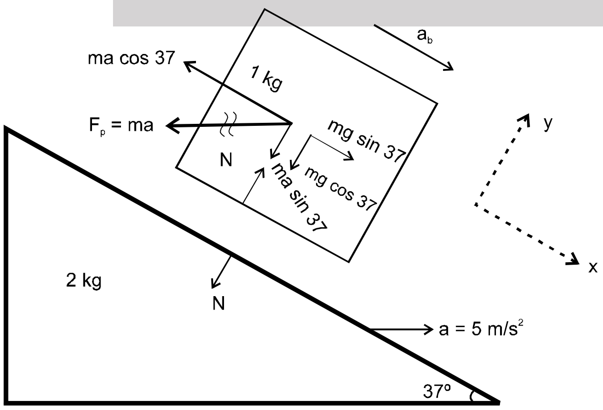
$F = m_1a_1 + m_2a_2$ [Newton's law for system]
$200 = 10 \times 12 + 20 \times a$ $a = 4$ m/s^2.

G-2.



$F = m1a1$ [Newton's II law for m1]
$180 = 20\ a_1$
$\Rightarrow$ $a_1 = 9$ m/s^2
Net force on m_2 is 0 therefore acceleration of m_2 is 0.

SECTION (H)
H-1.



FBD of block is shown w.r.t. wedge and FBD of wedge is shown w.r.t. ground. F_P is pseudo force.
$mg \sin 37 - ma \cos 37 = ma_b$
$\Rightarrow$ $a_b = g \sin 37 - a \cos 37 = 10 \times 3/5 - 5 \times 4/5 = 2$ m.s^2 w.r.t. wedge

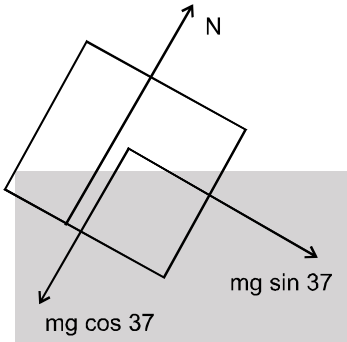
⇒ Block is not stationary w.r.t. wedge

N – ma sin 37 – mg cos 37 = 0 [Newton's II law for block]

⇒ N = 1 × 10 × 4/5 + 1 × 5 × 3/5

⇒ N = 11 N.

Net force acting on block w.r.t. ground.

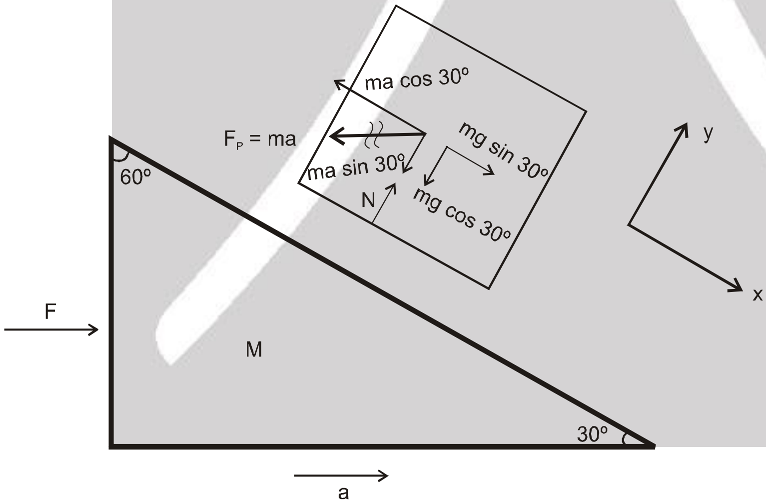


$$F = \sqrt{(mg\ \sin\ 37)^2 + (mg\ \cos\ 37 - N)^2}$$

$$= \sqrt{\left(10 \times \frac{3}{5}\right)^2 + \left(10\frac{4}{5} - 11\right)^2} = \sqrt{6^2 + 3^2}$$

$F = 3\sqrt{5}$ N.

H-2.



F.B.D. of wedge is w.r.t. ground and

F.B.D. of block is w.r.t. wedge.

Let a is the acceleration of wedge due to force F.

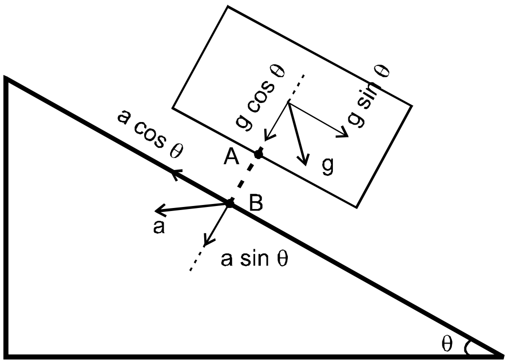
F_P is pseudo force on block

mg sin 30º – ma cos 30º = 0 [Equilibrium of block in x direction w.r.t. wedge] a = g tan 30º

F = (M + m)a [Newtons II law for the system of block and wedge in horizontal direction]

⇒ F = (M + m) g tan 30º.

H-3.



Acceleration of point A and B must be some along the line ⊥ to the surface

⇒ $a \sin\theta = g \cos\theta$

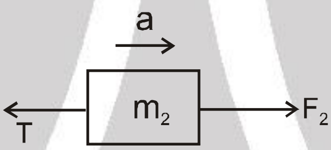
$a = g \cot\theta$

PART - III

1. Let a be acceleration of two block system towards right

$\therefore\ a = \dfrac{F_2 - F_1}{m_1 + m_2}$

The F.B.D. of m_2 is



$\therefore\quad F_2 - T = m_2 a$

Solving $T = \dfrac{m_1\ m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} + \dfrac{F_1}{m_1}\right)$

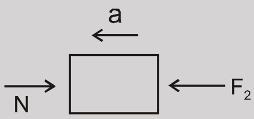
(B) Replace F_1 by $-F_1$ is result of A

$\therefore\quad T = \dfrac{m_1\ m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} - \dfrac{F_1}{m_1}\right)$

(C) Let a be acceleration of two block system towards left

$\therefore\quad a = \dfrac{F_2 - F_1}{m_1 + m_2}$

The FBD of m_2 is

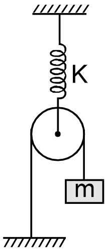


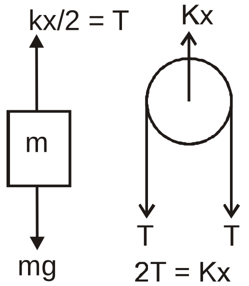
$\therefore\ F_2 - N_2 = m_2 a$

Solving $N = \dfrac{m_1\ m_2}{m_1 + m_2}\left(\dfrac{F_1}{m_1} + \dfrac{F_2}{m_2}\right)$

(D) Replace F_1 by $-F_1$ in result of C

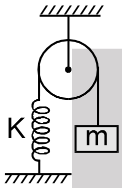
$N = \dfrac{m_1\ m_2}{m_1 + m_2}\left(\dfrac{F_2}{m_2} - \dfrac{F_1}{m_1}\right)$

2. (a) 

F.B.D. of m ; 

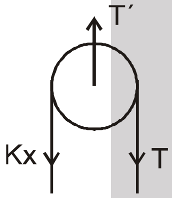
$2T = Kx$

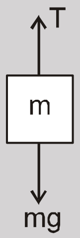
$\frac{kx}{2} = mg$ $\qquad x = \frac{2mg}{k}$ $\qquad T = mg$

(b) 

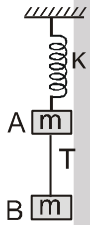
$kx - mg = 0$

F.B.D of pulley

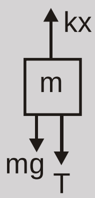




$x = \frac{mg}{k}$ $\qquad T = kx$ $\qquad \therefore\ T = mg\ ,\ T' = kx + T = 2mg$

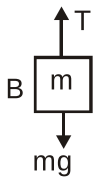
(c) 

F.B.D. Block A



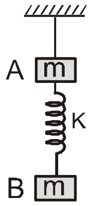
$kx = mg + T$ ....(1)

F.B.D. Block B

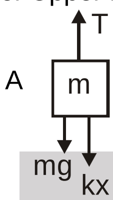


$T = mg$ ....(2)

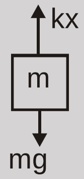
$\therefore\ kx = 2mg$ $\qquad x = \frac{2mg}{k}$

(d)


F.B.D. of Upper Block A



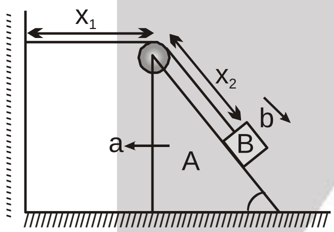
$T = mg + kx$ .... (1)

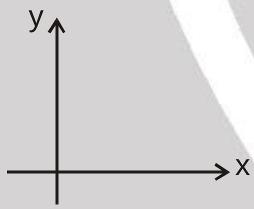
F.B.D. of Lower Block (B)


$kx = mg$ ..... (2) $\therefore x = \dfrac{mg}{k}$

By (1) & (2) $\quad T = 2mg$

3.





(a) Let b be acceleration of block B w.r.t. wedge

i.e. $\left|\vec{a}_{BW}\right| = b \quad \vec{a}_{BW} = b\cos\theta\,\hat{i} - b\sin\theta\,\hat{j}$

$\ell = x_1 + x_2$ ……..(1)

$\Rightarrow \quad 0 = \dfrac{dx_1}{dt} + \dfrac{dx_2}{dt} \quad \Rightarrow \; 0 = -a + b$

$\Rightarrow \quad b = a$ ……..(2)

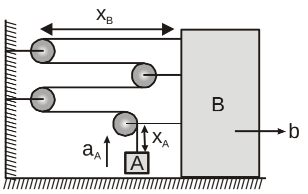
$\therefore \quad \vec{a}_{BW} = a\cos\theta\,\hat{i} - a\sin\theta\,\hat{j}$

$\vec{a}_{WG}$ = acceleration of wedge w.r.t. ground = $-a\,\hat{i}$ .... (3)

$\vec{a}_{BG} = \vec{a}_{BW} + \vec{a}_{WG}$

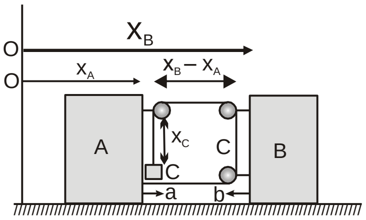
$\therefore \; \vec{a}_{BG} = (a\cos\theta - a)\,\hat{i} - a\sin\theta\,\hat{j}$ **Ans.**

(b)



$\ell = 4x_B + x_A \Rightarrow 0 = 4\dfrac{d^2x_B}{dt^2} + \dfrac{d^2x_A}{dt^2}$; $\dfrac{d^2x_A}{dt^2} = -a_{AB}$; $\dfrac{d^2x_B}{dt^2} = b \Rightarrow 4b = a_{AB}$

(c)



$\vec{a}_{CA} = \frac{d^2x_c}{dt^2}$; $a = \frac{d^2x_A}{dt}$, $b = -\frac{d^2x_B}{dt^2}$

Length = $x_C + x_B - x_A + C + x_B - x_A$

$\Rightarrow \quad \ell = x_C + 2x_B - 2x_A + C$

$\Rightarrow 0 = \frac{d^2x_C}{dt^2} + 2\frac{d^2x_B}{dt^2} - 2\frac{d^2x_A}{dt^2}$

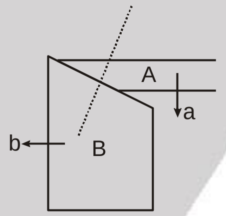
$\Rightarrow 0 = a_{CA} - 2b - 2a \qquad \therefore \vec{a}_{CA} = -(2a + 2b)$

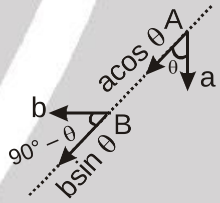
$\vec{a}_{CG} = \vec{a}_{CA} + \vec{a}_{AG} = -(2a + 2b)\,\hat{j} + a\,\hat{i}$

$\therefore \vec{a}_{CG} = a\,\hat{i} - 2(a+b)\,\hat{j}$ **Ans.**

(d) Let a be acceleration of wedge A.

Acceleration of blocks A & B along normal to contact surface (shown by dotted line) must be equal.



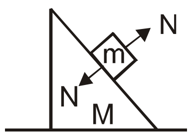


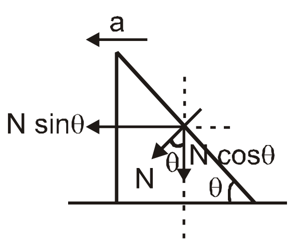
i.e. $b \sin\theta = a \cos\theta \qquad a = b \tan\theta$

$\therefore \vec{a}_A = -b \tan\theta\,\hat{j}$ **Ans.**

EXERCISE-2
PART - I

1.





$a = \frac{N\sin\theta}{M}$ along (– ve x axis)

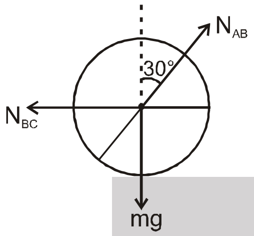
2. The free body diagram of cylinder is as shown.

Since net acceleration of cylinder is horizontal,

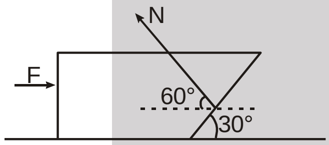
$N_{AB}\cos 30° = mg$ or $N_{AB} = mg$ .... (1)

and $N_{BC} - N_{AB}\sin 30° = ma$ or $N_{BC} = ma + N_{AB}\sin 30°$ .... (2)

Hence N_{AB} remains constant and N_{BC} increases with increase in a.



3.

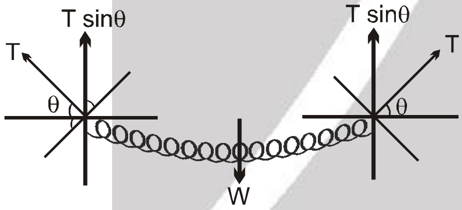


Acceleration of two mass system is $a = \frac{F}{2m}$ leftward

FBD of block A

$N\cos 60° - F = ma = \frac{mF}{2m}$ solving N = 3 F

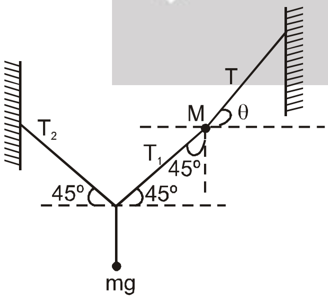
4.



$2T\sin\theta = W$

$T = W/2 \text{ cosec } \theta$

5.



$T_1\cos 45° = T_2\cos 45°$

$\Rightarrow$ $T_1 = T_2$

$(T_1 + T_2)\sin 45° = mg$

$\sqrt{2}\, T_1 = mg$

$T_1 = \frac{mg}{\sqrt{2}}$.

$$T\sin\theta = Mg + \frac{T_1}{\sqrt{2}}$$

$$T\sin\theta = Mg + \frac{mg}{2} \qquad ....(i)$$

$$T\cos\theta = \frac{T_1}{\sqrt{2}} = \frac{mg}{2} \qquad ....(ii)$$

Dividing (i) and (ii)

$$\tan\theta = \frac{M + m/2}{m/2} = 1 + \frac{2M}{m} \quad \text{Ans.}$$

6. $T = mg$

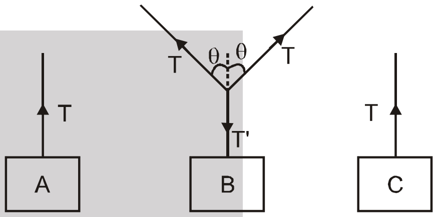
$2T\cos\theta = T'$

$T' = Mg$

$2mg\cos\theta = Mg$

$$\cos\theta = \frac{M}{2m} < 1$$

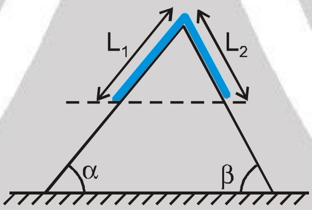
$M < 2m$



7. let L_1 and L_2 be the portions (of length) of rope on left and right surface of wedge as shown

∴ Magnitude of acceleration of rope

$$a = \frac{\frac{M}{L}[L_1\sin\alpha - L_2\sin\beta]\ g}{M} = 0 \qquad (\because L_1\sin\alpha = L_2\sin\beta)$$



8. By setting string length constant

$L = \ell_1 + 2\ell_2 + 2\ell_3$

After differentiation L' = 0 so

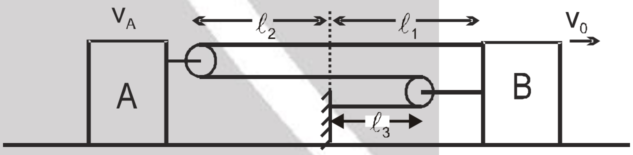
$-2v_A + v_0 + 2v_0 = 0$

⇒ $3v_0 = 2v_A$

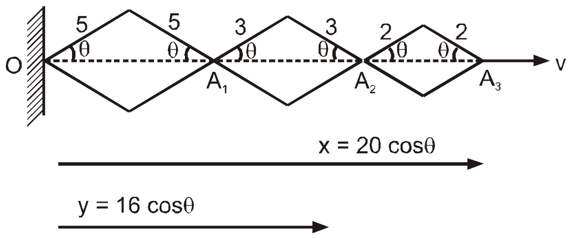
$$v_A = \frac{3}{2}v_0$$

$v_{AB} = v_A - v_B$

$$= \frac{v_0}{2} \text{ towards right.}$$

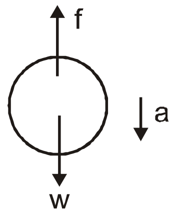


9.



$$v = \frac{dx}{dt} = -20\sin\theta\frac{d\theta}{dt} \quad \Rightarrow \quad u = \frac{dy}{dt} = -16\sin\theta\frac{d\theta}{dt} \quad \Rightarrow \quad u = \frac{4}{5}v = 0.8\ v$$

10.



$w - f = ma \qquad w - ma = g$

$w\left\{1 - \frac{m}{w}a\right\} = f \quad w\left\{1 - \frac{m}{mg}a\right\} = f \qquad w\left\{1 - \frac{a}{g}\right\} = f$

11. Legnth of groove = $\sqrt{3^2 + 4^2}$ = 5m

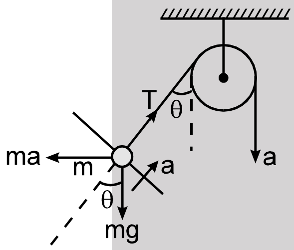
Acceleration along the incline = gsinθ = g sin 30º = g/2

Acceleration along the groove = g/2 cos (90–α) = g/2 sinα = $\frac{g}{2} \times \frac{4}{5}$ = 4m/s^2

$v^2 = 2as$

$v = \sqrt{2 \times 4 \times 5} = \sqrt{40}$ m/sec.

12.



(Force diagram in the frame of the car)

Applying Newton's law perpendicular to string

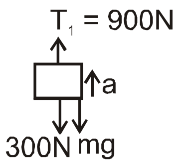
$mg \sin\theta = ma \cos\theta$

$\tan\theta = \frac{a}{g}$

Applying Newton's law along string

$\Rightarrow \qquad T - m\sqrt{g^2 + a^2} = ma \qquad$ **$T = m\sqrt{g^2 + a^2} + ma$ Ans.**

13.

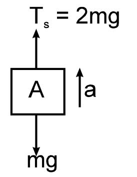


$900 - 300 - m \times 10 = ma \qquad 600 = m(10 + a)$

$\frac{600}{10 + a} = m$

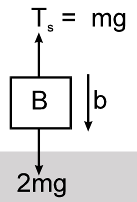
$\frac{600}{10 + 10} = m = \frac{600}{20} = 30$ kg.

14. For first case tension in spring will be



$T_s = 2mg$ just after 'A' is released.

$2mg - mg = ma \Rightarrow a = g$



In second case $T_s = mg$

$2mg - mg = 2mb$

$b = g/2$

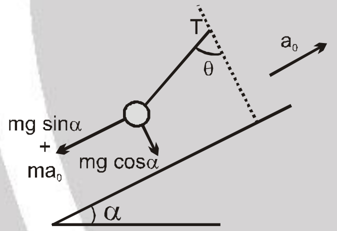
$a/b = 2$

15. $T \sin\theta = m(g \sin\alpha + a_0)$

$T \cos\theta = mg \cos\alpha$

$\Rightarrow \tan\theta = \left(\dfrac{g\sin\alpha + a_0}{g\cos\alpha}\right)$

$\theta = \tan^{-1}\left(\dfrac{g\sin\alpha + a_0}{g\cos\alpha}\right)$



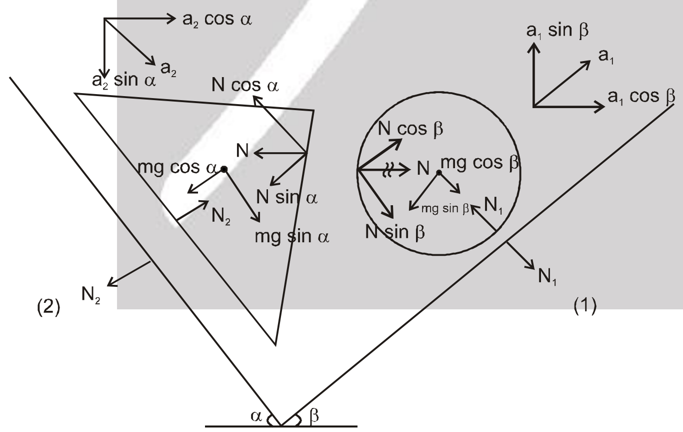
16. Slope of v_{rel} – t curve is Constant.

$\Rightarrow a_{rel} = \text{Const.} = a_1 - a_2 \neq 0$

Inference that at least one reference frame is accelerating both can't be non - accelerating simultaneously.

PART - II

1.



It is obvious that aceleration of cylinder is II to the wedge I and acceleration of triangular block is II to the wedge 2.

$a_2 \cos\alpha = a_1 \cos\beta$ [constrained relation between the contact surface of block and cylinder]

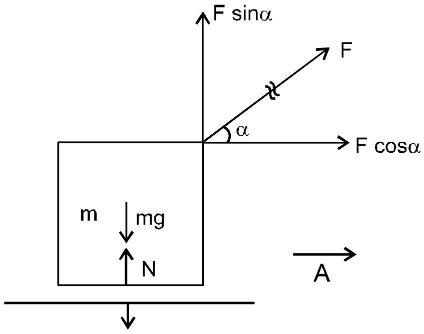
$N \cos\beta - m_1 g \sin\beta = m_1 a_1$ [Newton's II law for cylinder along the direction parallel to the wedge1]

$m_2 g \sin\alpha - N\cos\alpha = m_2 a_2$ [Newton's II law for block along the direction parallel to the wedge2]

By solving equation I, II and III we get

$$N = mg\left(\frac{\sin\alpha\cos\alpha + \sin\beta\cos\beta}{\cos^2\alpha + \cos^2\beta}\right) = 5N \text{ Ans}$$

2.



$mg - N - F\sin\alpha = 0$ [Equilibrium of block in vertical direction]

at breaking off the contact N = 0 .

$\Rightarrow \quad F\sin\alpha = mg$

$\Rightarrow \quad at\sin\alpha = mg$

$\Rightarrow \quad t = \frac{mg}{a\sin\alpha}$

$F\cos\alpha = mA$ [Newton's second law for block in horizontal direction]

$\Rightarrow \quad at\cos\alpha = m\frac{d\upsilon}{dt}$

$$\int_0^{\upsilon} d\upsilon = \frac{a\cos\alpha}{m}\int_0^{t=\frac{mg}{a\sin\alpha}} t\,dt$$

$$\Rightarrow \quad \upsilon = \frac{a\cos\alpha}{m}\frac{t^2}{2} \quad ....1$$

After putting time limits $\upsilon = \frac{mg^2\cos\alpha}{2a\sin^2\alpha}$

equation I can be written as $\frac{dx}{dt} = \frac{a\cos\alpha}{2m}t^2$

$$\int_0^x dx = \frac{a\cos\alpha}{2m}\int_0^t t^2\,dt = \frac{a\cos\alpha}{2m}\frac{t^3}{3}$$

After putting limits. $x = \frac{m^2g^3\cos\alpha}{6a^2\sin^3\alpha}$

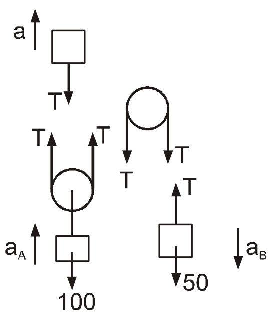
3. $v_{nm} = \frac{v_B + v_A/2}{2} = \frac{4 + 4/2}{2} = \frac{4+2}{2} = 3$

4. $a_A = \frac{d^2y}{dt^2} = \frac{1}{2}$

$a_B = 8a_A$ by constrained relation

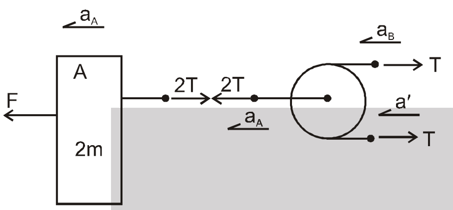
$a_B = 4\ m/s^2$

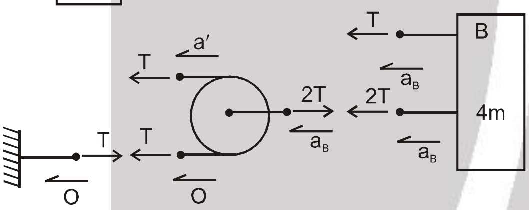
5.



$2a_A = a + a_B$
$2a_A = 3 + a_B$
$2T – 100 = 10a_A$
$50 – T = 5a_B$
$\Rightarrow$ $a_B + a_A = 0$
$2a_A – 3 + a_A = 0$
$a_A = 1 m/s^2$
$\Rightarrow$ $a_B = – 1 m/s^2$.

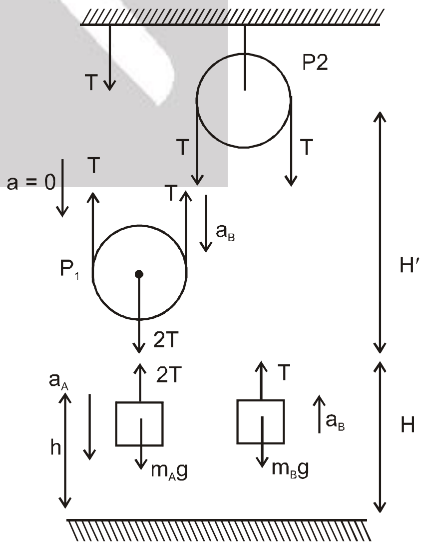
6.





$a_B + a' = 2a_A$ [constrained relation for pulley 1]
$O + a' = 2a_B$ [contrained relation for pulley 2]
From above two equations
$3a_B = 2a_A$
$\Rightarrow a_A = \frac{3}{2} a_B$ I
$F – 2T = 2ma_A$ [Netwon's II law for block A] II
$3T = 4 m a_B$ [Netwon's II law for block B] III
From equation I, II and III
$a_B = \frac{3F}{17m}$.

7. $m_Ag – 2T = m_Aa_A$ [Newton's II law for block A]
$T – m_Bg = m_Ba_B$ [Newton's II law for block B]
$a_B + O = 2a_A$ [constrained relation for pulley P1]
$m_A = 4m_B$ [Given in question]
From above four equations



$a_A = \frac{g}{4} = 2.5 m/s^2$
$a_B = \frac{g}{2} = 5 m/s^2$
$h = \frac{1}{2} a_At^2$ [Equation of motion for block A]
$\Rightarrow$ $t = \frac{2}{5}$ sec.
H is the distance travelled by block B in vertical direction till $\frac{2}{5}$ second
$\Rightarrow$ $H = \frac{1}{2} a_Bt^2$ [Equation of motion for block B]

$\Rightarrow \quad \frac{1}{2} 5 \left(\frac{2}{5}\right)^2$

H = 0.4 m

H´ is the distance travelled by block B due to gained velocity.

$v_1 = a_B t$

= 5 × 0.4

v_1 = 2 m/s

$v_2^2 = v_1^2 + 2a H´$

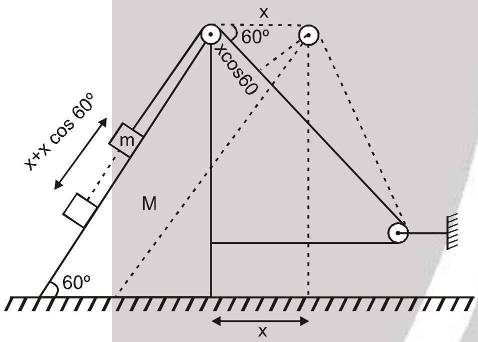
$0^2 = 2^2 + 2(-10) H´$

$H´ = \frac{2}{10} = 0.2$ m

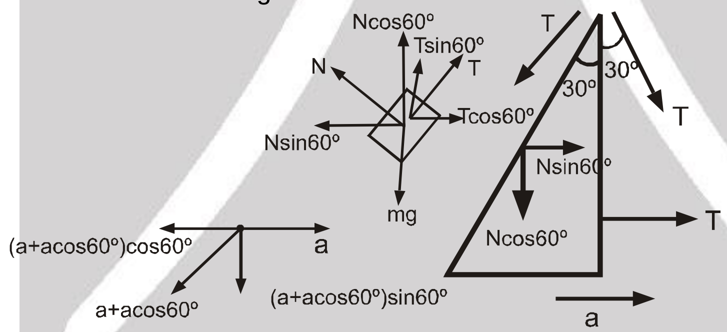
Total distance = H + H´

= 0.6 m = 60 cm.

8.



$\Rightarrow$ If acceleration of wedge is x then acceleration of block w.r.t. wedge is x + x cos60º.



T + N sin60º = Ma

$T + N\frac{\sqrt{3}}{2} = Ma$

T cos60º – N sin60º = m[a – a cos60º – a cos²60º]

$\frac{T}{2} - \frac{N\sqrt{3}}{2} = ma\left[1 - \frac{1}{2} - \frac{1}{4}\right]$

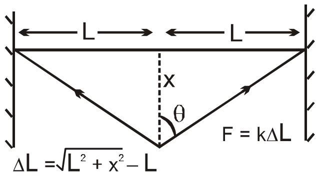
$\Rightarrow \quad T - N\sqrt{3} = \frac{ma}{2}$

mg – N cos60º – T sin60º = m(a sin60º + a cos60º sin60º)

$mg - \frac{N}{2} - \frac{T\sqrt{3}}{2} = ma\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{4}\right]$

$2mg - N - T\sqrt{3} = \frac{3\sqrt{3}}{2}\, ma \qquad \Rightarrow \qquad a = \frac{30\sqrt{3}}{23}$ m/s².

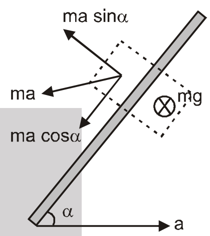
9.



$F_{net_{कुल}} = mg - 2F\cos\theta$

$$a_{net} = g - \frac{2k}{m}\left(\sqrt{L^2+x^2} - L\right)\frac{x}{\sqrt{L^2+x^2}}$$

10. Acceleration of bead along rod is

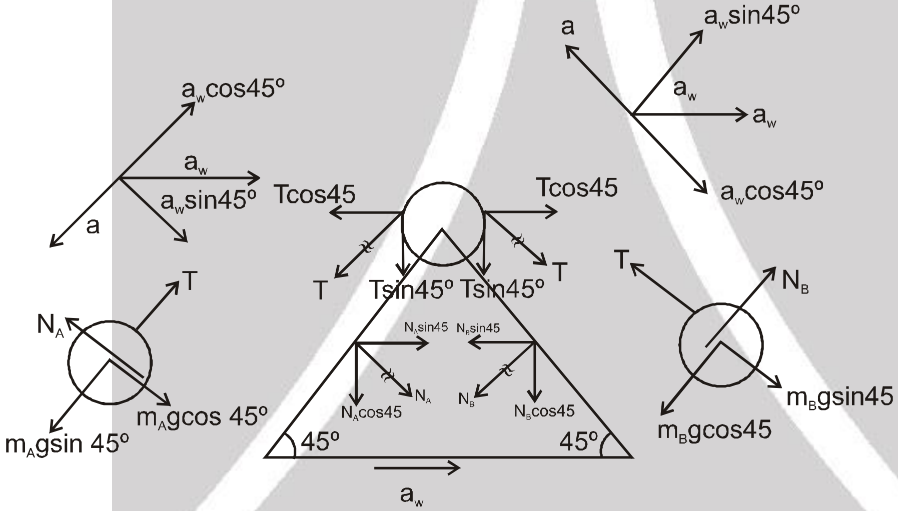


$\frac{ma\cos\alpha}{m} = a\cos\alpha$

$\frac{1}{2}a\cos\alpha t^2 = \ell$

$t = \sqrt{\frac{2\ell}{a\cos\alpha}} = 2\text{sec}$

11.



All the forces shown are in ground frame. a_w is the acceleration of wedge w.r.t ground and a is the acceleration of blocks w.r.t wedge.

$m_A g\sin45^o - T = m_A(a - a_w\cos45^o)$ [Newton's II law for block A along the wedge in ground frame]

$m_A g\cos\theta - N = m_A a_w\sin45^o$ [Newton's Ii law for block A in direction ⊥ to the wedge in ground frame.]

$T - m_B g\sin45 = m_B(a - a_w\cos45)$ [Newton's II law for block B along the wedge in ground frame.]

$N_B - m_B g\cos45^o = m_B(a_w\sin45)$ [Newton's II law for block B in direction ⊥ to the wedge in ground frame]

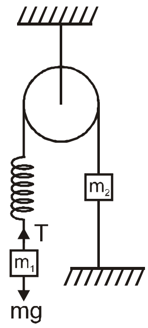
$N_A\sin45 + T\cos45 - N_B\sin45 - T\cos45 = m_w a_w$

[Newton's II law for wedge in horizontal direction in ground frame].

After solving above five equations we will get $a_w = \frac{2}{5}\text{ m/s}^2 = 40\text{ cm/s}^2$

PART - III

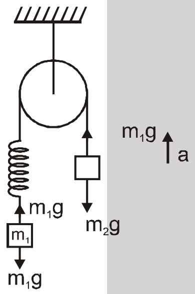
1.



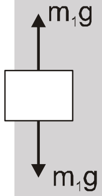
$T = m_1g$

when thred is burnt, tension in spring remains same = m_1g.

$m_1g - m_2g = m_2a$ $\qquad \frac{(m_1 - m_2)}{m_2} g = a$ = upwards



for m_1



$a = o$

2. $F = \alpha t$

$a = \frac{dv}{dt} = \frac{\alpha}{m} t$ ....(i) straight line curve 1

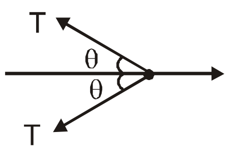
$dv = \frac{\alpha}{m}\ tdt$

$v = \frac{\alpha}{m} \frac{t^2}{2}$ curve 2 ...(ii)

divide (ii) by (ii)

$v = \frac{t}{2} a = \frac{a}{2} \times \frac{am}{\alpha} = \frac{a^2 m}{2\alpha}$ $\rightarrow$ Paacebole curve 2.

3.



$F = 2\ T \cos\theta \quad T = \frac{F}{2\cos\theta}$

$\theta\uparrow \cos\theta\downarrow T\uparrow$

on incrcasing θ, $\cos\theta$ decreases and hence T increases.

4. By string constraint

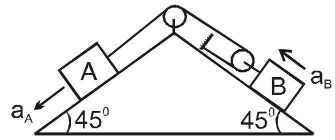
$a_A = 2a_B$ ……..(1)

Equation for block A.

$10 \times 10 \times \frac{1}{\sqrt{2}} - T = 10\, a_A$ ……..(2)

Equation for block B.

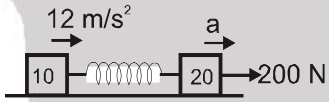
$2T - \frac{400}{\sqrt{2}} = 40\, a_B$ ……..(3)



Solving equation (1), (2) & (3), we get $a_A = \frac{-5}{\sqrt{2}}$ m/s^2

$a_B = \frac{-5}{2\sqrt{2}}$ m/s^2 $\Rightarrow$ $T = \frac{150}{\sqrt{2}}$ N

5. Apply NLM on the system



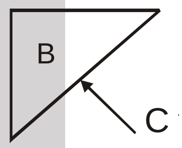
$200 = 20\, a + 12 \times 10$

$\frac{80}{20} = a$

$= 4$ m/s^2

Spring Force $= 10 \times 12 = 120$ N

6. There is no horizontal force on block A, therefore it does not move in x-directing, whereas there is net downward force (mg – N) is acting on it, making its acceleration along negative y-direction.

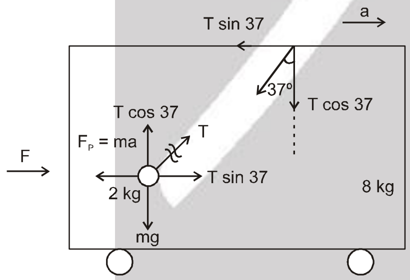


Block B moves downward as well as in negative x-direction. Downward acceleration of A and B will be equal due to constrain, thus w.r.t. B, A moves in positive x-direction.

Due to the component of normal exerted by C on B, it moves in negative x-direction.

7. Pseudo force depends on acceleraton of frame and mass of object

8.



F.B.D. of trolley is w.r.t. ground

F.B.D. of suspended mass is w.r.t. Trolley.

$T\cos 37^o - mg = 0$ [Equilibrium of mass in y direction w.r.t. trolley]

$\Rightarrow$ $T = \frac{5\ mg}{4}$ $T = 25$ N

$T \sin 37^o - ma = 0$ [Equilibrium of mass in x direction w.r.t. trolley]

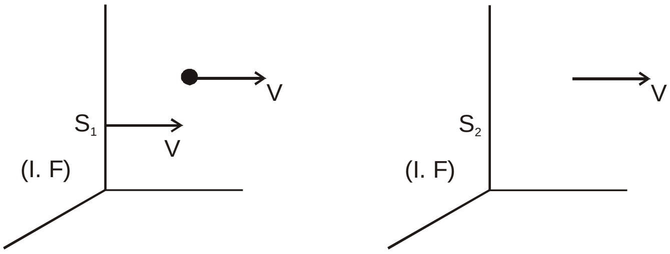
$\Rightarrow$ $a = \frac{T \sin 37}{m} = \frac{15}{2}$

$F - T \sin 37 = 8a$ [Newton's II law for trolley in x direction w.r.t. ground]

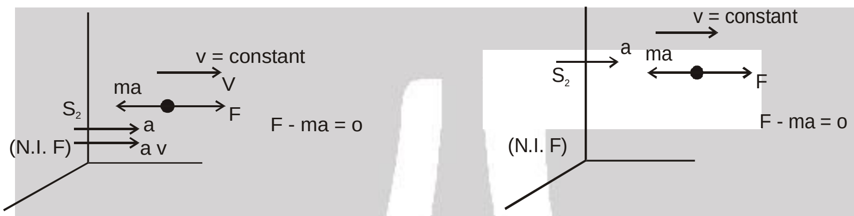
$\Rightarrow$ $F = 8 \times 15/2 + 25 \times 3/5$ $F = 75$ N

9. (A) True

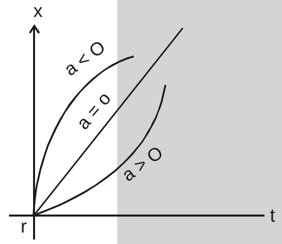
(B) True



Accelerated & moving with velocity V. Accelerated but not moving.



10.



x -t curve is

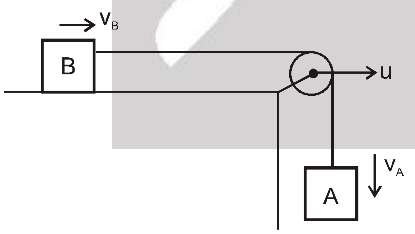
(1) straight line when a = 0

(2) concave up when uniform a > 0

(3) concave down when uniform a < 0.

In the region AB & CD acceleration = 0 = Force = 0

11.



By string constrain

$v_A + u - v_B = 0$

or $v_B = u + v_A$

Differentiating both side

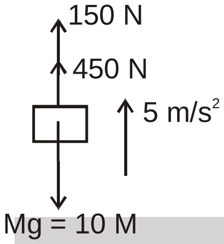
$a_B = 0 + a_A$ **Ans.**

PART - IV

1. FBD of Block in ground frame :

Applying N.L. $150 + 450 - 10\,M = 5M$

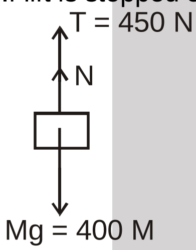
$\Rightarrow$ $15\,M = 600$ $\Rightarrow$ $M = \frac{600}{15}$



$\Rightarrow$ M = **40 Kg Ans.**

Normal on block is the reading of weighing machine i.e. 150 N.

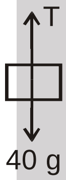
2. If lift is stopped & equilibrium is reached then



$450 + N = 400$

$\Rightarrow$ $N = -50$

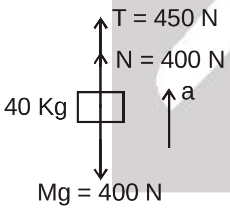
So block will lose the contact with weighing machine thus reading of weighing machine will be zero.



$T = 40\,g$

So reading of spring balance will be 40 Kg.

3.



$a = \frac{950 - 400}{40}$ $\Rightarrow$ $a = \frac{450}{40} = \frac{45}{4}\,m/s^2$ **Ans.**

4. $a_p = \frac{10\ \ t}{10} = t$

$\therefore$ $\frac{dv}{dt} = t$ $\Rightarrow$ $\int_0^v dv = \int_0^t t\ \ dt$ $\Rightarrow$ $v = \frac{t^2}{2}$

Putting v = 2 we have t = 2 sec.

Now $\frac{dx}{dt} = \frac{t^2}{2}$ $\therefore$ $x_p = \left[\frac{t^3}{6}\right]_0^2 = \frac{4}{3}$

$x_B = 2 \times 2 = 4$ m

Hence relative displacement = 4 – 4/3 = 8/3 m

5. From above

$2t = t^3/6 \quad \Rightarrow \quad t^2 = 12 \quad \Rightarrow t = 2\sqrt{3}$ sec.

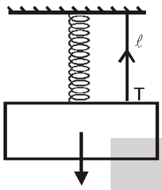
6. $a = t = 4$

$\therefore$ after 4 seconds $V_B = 2$ m/s

$V_p = 4^2/2 = 8$ m/s

$\therefore$ $V_{rel} = 8 - 2 = 6$ m/s.

9.



(i) $\Delta\ell = \ell/2$

$F_s = K\Delta\ell$

$< \frac{2mg}{\ell}\frac{\ell}{2}$

$F_s < mg$

$T + F_s = mg$

$T = mg - \frac{K\ell}{2}$

(ii) $mg - \frac{K\ell}{2} = ma$

$g - \frac{k\ell}{2m} = a$

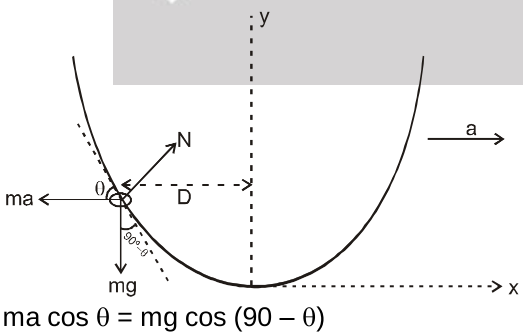
If it is so

$F_s > mg$

i.e., $\Delta\ell < \frac{\ell}{2}$ string unstretched & $T = 0$.

EXERCISE-3
PART - I

1.



$ma \cos\theta = mg \cos(90 - \theta)$

$\Rightarrow \frac{a}{g} = \tan\theta \quad \Rightarrow \quad \frac{a}{g} = \frac{dy}{dx}$

$\Rightarrow \frac{d}{dx}(kx^2) = \frac{a}{g} \quad \Rightarrow \quad x = \frac{a}{2gk} = D$

PART - II

1. Vertical component of acceleration of A

$a_1 = (g \sin\theta).\sin\theta$

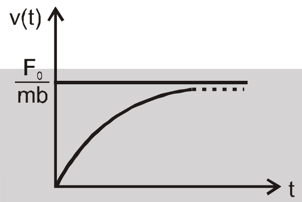
$= g \sin 60^\circ . \sin 60^\circ = g . 3/4$

That for B

$a_2 = g \sin 30^\circ . \sin 30^\circ = g \frac{1}{4}$

$\therefore \quad (a_{AB})_\perp = \frac{3g}{4} - \frac{g}{4} = \frac{g}{2} = 4.9 \text{ m/s}^2$

2. $F = ma = F_0 e^{-bt}$



$$\frac{dv}{dt} = \frac{F_0}{m} e^{-bt}$$

$$\int_0^v dv = \frac{F_0}{m}\int_0^t e^{-bt} dt \quad ; \quad v = \frac{F_0}{m}\left[\frac{e^{-bt}}{-b}\right]_0^t$$

$$v = \frac{F_0}{mb}\left(1 - e^{-bt}\right)$$

3. $a = -(g + \gamma v^2)$

$$\frac{dv}{dt} = -(g + \gamma v^2)$$

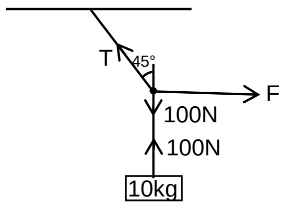
$$\int_{v_0}^{0} \frac{dv}{g + \gamma v^2} = -\int_0^t dt$$

$$\frac{1}{\gamma}\int_{v_0}^{0} \frac{dv}{\left(\frac{g}{\gamma} + v^2\right)} = -\int_0^t dt$$

$$\frac{1}{\gamma}\frac{1}{\sqrt{\frac{g}{\gamma}}}\left[\tan^{-1}\left(\frac{v}{\sqrt{\frac{g}{\gamma}}}\right)\right]_{v_0}^{0} = -t$$

$$\frac{1}{\sqrt{g\gamma}}\tan^{-1}\left(\frac{\sqrt{\gamma}}{\sqrt{g}} v_0\right) = t$$

4.

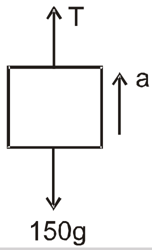


$\frac{T}{\sqrt{2}} = 100$; $\quad \frac{T}{\sqrt{2}} = F$; $\quad F = 100N.$

HIGH LEVEL PROBLEMS (HLP)

1. (a) (i) acceleration at t = 1s

$a = \dfrac{3.6-0}{2-0} = 1.8\ m/s^2$



$T - 150\,g = 150\,a$

$T = 150 \times 9.8 + 150 \times 1.8$

$= 1740$ N.

(ii) At t = 6s t = 6s, a = o

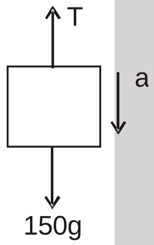
$\therefore$ T = 150 g N

$= 150 \times 9.8 = 1470$N

(iii) At t = 118 t = 118 ; $a = -1.8\ m/s^2$

$1.8\ m/s^2$ down

$150\,g - T = 150\,a$



$T = 150 \times (9.8 - 1.8) = 1200$ N

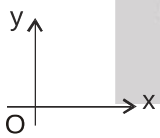
(b) Height = Area of v - t graph

$= 1/2(12 + 8)3.6 = 36$ m

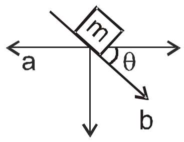
(c) Average velocity = $\dfrac{\text{Displacement}}{\text{time}} = \dfrac{36}{12} = 3$ m/s

(d) Average acceleration = $\dfrac{\text{change in velocity}}{\text{time in taken}} = \dfrac{0-0}{12} = 0$

2.



$\vec{a}_A = -a\,\hat{i}$



$\vec{a}_B = (b\cos\theta - a)\,\hat{i} - b\sin\theta\,\hat{j}$

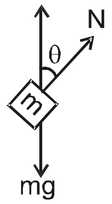
As there is no external force along x direction

$\therefore \quad 2ma_{Ax} + ma_{Bx} = 0$

$\Rightarrow \quad 2m(-a) + m\,(b\cos\theta - a) = 0$

$\Rightarrow \quad 3a = b\cos\theta$ .... (1)

$\therefore \quad \vec{a}_B = 2a\,\hat{i} - 3a\,\hat{j}\,\tan\theta$ .... (2)



$\therefore$ along x-direction

$N \sin \theta = m \times 2a$ ....(2)

Along y-direction

$mg - N\cos \theta = m\, 3a \tan \theta$ ....(3)

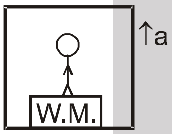
$\Rightarrow$ $mg - 2ma \cot \theta = 3ma \tan \theta$

$\Rightarrow$ $g = a\,[2\cot \theta + 3 \tan \theta]$

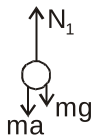
$$a = \frac{g\sin\theta\cos\theta}{2\cos^2\theta + 3\sin^2\theta}$$

$a = \dfrac{g\sin\theta\cos\theta}{3-\cos^2\theta}$ $\qquad$ $b = \dfrac{3g\sin\theta}{3-\cos^2\theta}$

3.

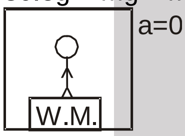


F.B.D. in NIF.

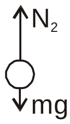


$N_1 = mg + ma$

$80.5g = mg + ma$ .... (1)

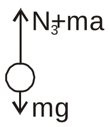


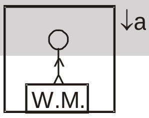
F.B.D. in



$N_2 = mg$ .... (2)

F.B.D. in N.I.F.





$N_3 + ma = mg$

$\Rightarrow N_3 = mg - ma$ $\qquad$ $\Rightarrow 59.5\,g = mg - ma$ .... (3)

(1) + (3) $\qquad$ $140\,g = 2mg$

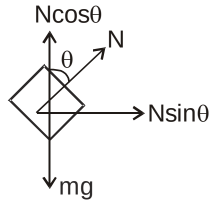
m = 70 kg Ans.

(a) $\therefore$ N_2 = true weight = **70 kg. Ans.**

(b) by (1) $80.5 \times g = mg + ma$

$\Rightarrow$ $10.5\,g = 70\,a$ $\qquad$ $\Rightarrow$ $a = \dfrac{10.5 \times 10}{70} = 1.5\ m/s^2$ **Ans.**

4. Let a be acceleration of system

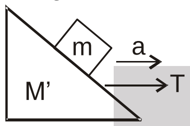


a → $N\sin\theta = ma$ ……(1)

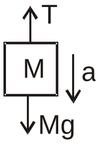
$N\cos\theta = mg$ ……(2)

Dividing (1) by (2) , we get

$a = g\tan\theta$ ……(3)



$T = (M' + m)\,a$ ……(4)

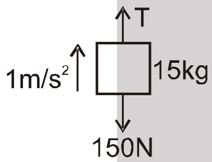


$Mg - T = Ma$ ……(5)

(4) + (5) $Mg = (M' + m + M)a$ ……(6)

by (3) & (6) $Mg = (M' + m + M)g\tan\theta$ $\Rightarrow M = \dfrac{M' + m}{\cot\theta - 1}$ **Ans.**

5.



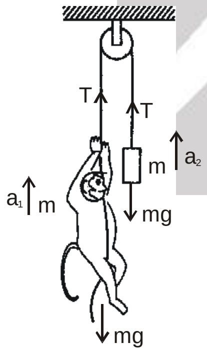
$T - 150 = 15 \times 1$

T = 165 N Ans.

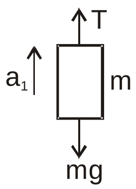
$S = \frac{1}{2}at^2$ $5 = \frac{1}{2} \times 1 \times t^2$

t = $\sqrt{10}$ s Ans.

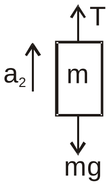
6. Let a_1 & a_2 be acceleration of monkey & Block respectively



F.B.D. Monkey



$ma_1 = T - mg$ $a_1 = \dfrac{T}{m} - g$ .... (1)

F.B.D. Block 

$ma_2 = T - mg \quad a_2 = \frac{T}{m} - g$ .... (2)

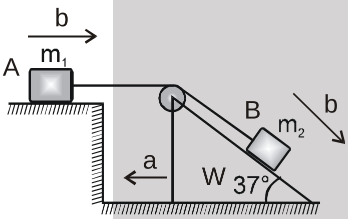
By (1) & (2)

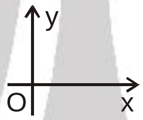
$a_1 = a_2 \qquad \therefore \qquad a_{rel} = 0$, as $u_{rel} = 0$

Relative displacement is zero.

Hence separation remains same.

7. Let b be acceleration of masses m_1 & m_2 with respect to wedge & a be acceleration of wedge w.r.t. ground.





$\vec{a}_{WG} = -a\hat{i}$ ...... (1)

$\vec{a}_{AG} = \vec{a}_{AW} + \vec{a}_{WG}$

$= b\hat{i} - a\hat{i} \Rightarrow \vec{a}_{AG} = (b-a)\,\hat{i}$ .... (2)

$\vec{a}_{BG} = \vec{a}_{BW} + \vec{a}_{WG} = b\cos 37°\,\hat{i} - b\sin 37°\,\hat{j} - a\,\hat{i}$

$\vec{a}_{BG} = \left(\frac{4b}{5} - a\right)\hat{i} - \frac{3b}{5}\hat{j}$ .... (3)

As $F_{external,\,x} = 0$

$\Rightarrow \quad M_A a_{AG,\,x} + M_B a_{BG,\,x} + m_W\, a_{WG,\,x} = 0$

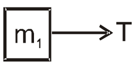
$\Rightarrow \quad 1.3\,(b-a) + 1.5\left(\frac{4b}{5} - a\right) + 3.45\,(-a) = 0$

$\Rightarrow \quad (1.3 + 1.5 + 3.45)\,a = (1.3 + 1.2)\,b$

$\Rightarrow \quad 6.25\,a = 2.5\,b$

$\Rightarrow \quad 5a = 2\,b$ ....(1)

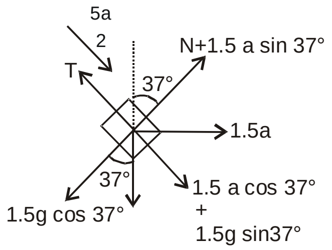
$b - a = \frac{3}{2}a$

F.B.D. System : m_1 

Frame : $T = 1.3 \times \frac{3}{2}a$ ....(2)

F.B.D. System : m_2

Frame :



Along the incline :

$$1.5a\frac{4}{5} + 1.5g\ \frac{3}{5} - T = 1.5\frac{5a}{2}$$

$\Rightarrow$ $9 - T = 2.55\ a$ ....(3)

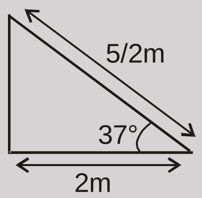
by (2) & (3) (2)

$9 = 4.5\ a$

$\Rightarrow$ $a = 2 m/s^2$

$\therefore$ $b = 5 m/s^2$

$S = \frac{1}{2}bt^2 \quad \Rightarrow \quad \frac{5}{2} = \frac{1}{2} \times 5t^2 \quad t = 1\ s$



(i) $\therefore\ V_{m_3} = u + a_{m_3} = 0 + 2 \times 1$

$\therefore\ V_{m_3} = 2 m/s$ **Ans.**

(ii) $\vec{a}_{BG} = \left(\frac{4}{5}\times 5 - 2\right)\hat{i} - \frac{3}{5} \times 5\,\hat{j}$

$\vec{a}_{BG} = 2\,\hat{i} - 3\,\hat{j}$

$a_{M_2} = |\vec{a}_{BG}| = \sqrt{13}\ m/s^2$

$V_{M_2} = a_{m_2}\ t$

$V_{M_2} = \sqrt{13}\ m/s^2$ **Ans.**

by (2) $T = \frac{3.9}{2} \times 2 \quad \Rightarrow T = 3.9$ N Ans.

8. $m > m'$

Let a be acceleration of M w.r.t. ground

b_1 = acceleration of m' w.r.t. ground

b_2 = acceleration of m w.r.t. ground

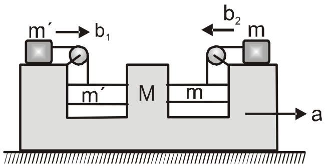
$\vec{a}_{MG} = a\,\hat{i} \qquad a_{m'G} = b_1\,\hat{i} \qquad \vec{a}_{mG} = -b_2\,\hat{i}$

As $F_{external\ x} = 0$

$\Rightarrow$ $m'a_{m'Gx} + (M + m + m')\ a_{MGx} + m\ a_{MG,\ x} = 0$

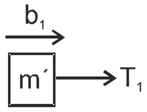
$m'b_1 + (M + m + m')a - mb_2$

$m'b_2 - m'b_1 = (M + m + m')a$ ....(1)



F.B.D. System : m'

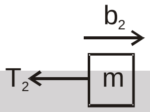
Frame : I.F.



$T_1 = m'b_1$ .... (2)

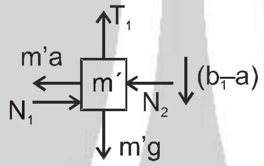
F.B.D. System : M

Frame :



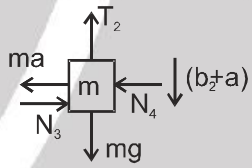
$T_2 = mb_2$ .... (3)

F.B.D.
System



Frame : N.I.F. $m'g - T_1 = m'(b_1 - a)$ .... (4)

F.B.D.
System :



Frame : N.I.F. $mg - T_2 = m(b_2 + a)$ .... (5)

(2) + (4) $\Rightarrow$ $m'g = m'(2b_1 - a)$

$g = 2b_1 - a$ .... (6)

(3) + (5) $\Rightarrow$ $mg = m(2b_2 + a)$ .... (7)

$g = 2b_2 + a$

Solving (1), (6) & (7) we get

$$a = \frac{(m - m')g}{2M + 3m + 3m'}$$ **Ans.**

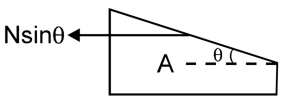
9. Let the acceleration of B downwards be $a_B = a$

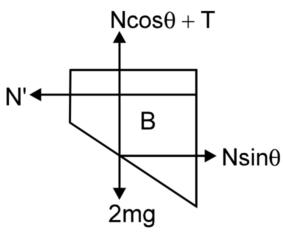
From constraint ; acceleration of A and C are

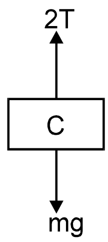
$$a_A = a \cot\theta = \frac{4a}{3} \text{ towards left}$$

$$a_C = \frac{a}{2} \text{ upwards}$$

free body diagram of A, B and C are







$$N \sin\theta = \frac{9m}{64}(a \cot\theta) \quad \text{........................(1)}$$

$2\,mg - T - N\cos\theta = 2ma$ ....(2)

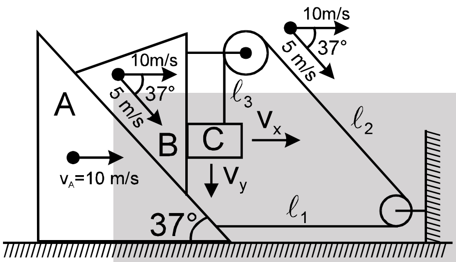
$2T - mg = m\frac{a}{2}$ ....(3)

solving we get

$$a_c = \frac{a}{2} = 3\text{m/s}^2$$

Ans. 3m/s^2 upwards

10.



Let v_x and v_y be the horizontal and vertical component of velocity of block C.

The component of relative velocity of B and C normal to the surface of contact is zero.

$\therefore \quad 10 + 5\cos 37° - v_x = 0$ ...(1)

$v_x = 14$ m/s

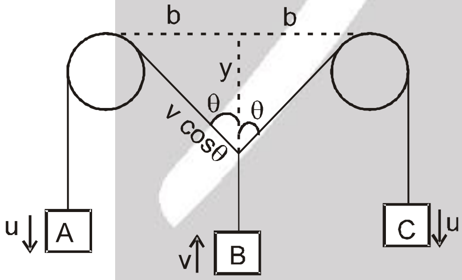
From the figure $\ell_1 + \ell_2 + \ell_3$ = constant

$$\therefore \quad \frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} = 0$$

$(-10) + (-5 - 10\cos 37°) + (-5\sin 37° + v_y) = 0 \quad \therefore \quad v_y = 26$ m/s.

11. Pseudo force on a particle depends on mass of particle and negative accleration of observer.

12.



$v\cos\theta = u$

$v = u\sec\theta$

$$\frac{dv}{dt} = u\sec\theta\tan\theta\frac{d\theta}{dt} \text{ .... I}$$

$\tan\theta = b/y$

$$\sec^2\theta\frac{d\theta}{dt} = -\frac{b}{y^2}\frac{dy}{dt}$$

$$= +\frac{b}{y^2}\cos^2\theta\frac{u}{\cos\theta}$$

$$= \frac{1}{b}\frac{b^2}{y^2}\cos\theta\, u$$

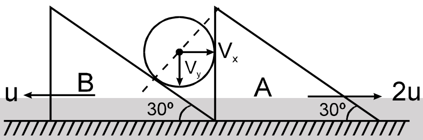
$= \frac{u\cos\theta}{b}\tan^2\theta$ II

$\Rightarrow \frac{dv}{dt} = \frac{u^2}{b}\tan^3\theta$ from I and II $\Rightarrow \frac{dv}{dt} = \frac{u^2}{b}\tan^3\theta$

13. **Method - I**

As cylinder will remains in contact with wedge A

$V_x = 2u$



As it also remain in contact with wedge B

$u \sin 30° = V_y \cos30° - V_x \sin30°$

$V_y = V_x \frac{\sin30°}{\cos30°} + \frac{U\sin30°}{\cos30°}$

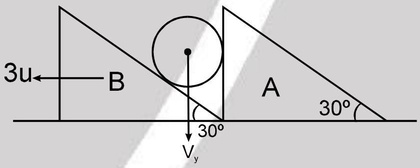
$V_y = V_x \tan30° + u \tan 30°$

$V_y = 3u \tan30° = \sqrt{3}\,u$

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}\,u$ **Ans.**

Method - II

In the frame of A

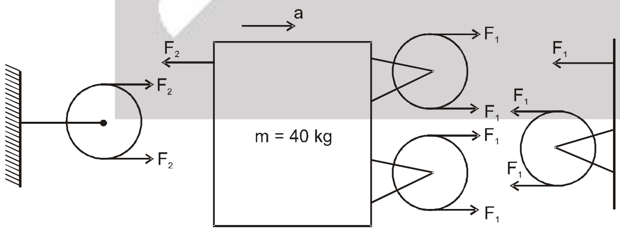


$3u \sin 30^o = V_y\cos30^o$

$\Rightarrow$ $V_y = 3u \tan 30^o = \sqrt{3}\,u$

and $V_x = 2u$ $\Rightarrow$ $V = \sqrt{V_x^2 + V_y^2} = \sqrt{7}u$ **Ans.** .

14.



$4F_1 - F_2 = ma$ [Newtons II law for block]

$\Rightarrow a = \frac{4F_1 - F_2}{m}$

t = 0 to 2 sec.

$F_1 = 30N$

$F_2 = 10N$

$\Rightarrow a = \frac{4\times30-10}{40} = 2.75 \text{ m/s}^2$

t = 2 to 4 sec

$F_1 = 30N$

$F_2 = 20N$

$\Rightarrow a = \dfrac{4 \times 30 - 20}{40} = 2.5\ m/s^2$

For t = 4 to 6 sec.

$F_1 = 10N$

$F_2 = 40N$

$\Rightarrow a = \dfrac{4 \times 10 - 40}{40} = 0\ m/s^2$

For t = 6 to 12 sec

$F_1 = 0$, $F_2 = 0$

$\Rightarrow a = 0\ m/s^2$

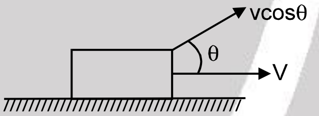
$V_{12} - V_0 = a_{0-2}(2 - 0) + a_{2-4}(4 - 2) + a_{4-6}(6 - 4) + a_{6-12}(12 - 6)$

$V_{12} - 1.5 = 2.75 \times 2 + 2.5 \times 2 + 0 \times 2 + 0 \times 6$

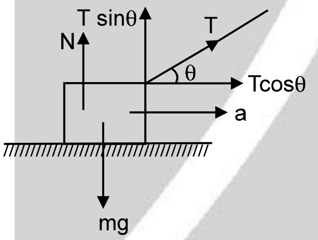
$V_{12} = 12\ m/s$

15. By constraint velocity component of block along the string should be u

$\Rightarrow$ $v \cos\theta = u$ or $v = u \sec\theta$ ....(1)



from (1) $a = \dfrac{dv}{dt} = u \sec\theta \tan\theta \dfrac{d\theta}{dt}$ ....(2)



Initially when block is at a large distance θ is a small component of T in vertical direction is very small. As block comes nearer and nearer. T sin θ increases and N decreases.

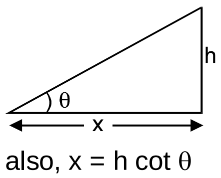
When T sinθ = mg then block just loses contact with the ground

so $T \sin\theta = mg$ ....(3)

$T \cos\theta = ma$ ....(4)

(3) & (4) $\Rightarrow$

$a \tan\theta = g$ ....(5)



also, $x = h \cot\theta$

$\dfrac{dx}{dt} = -h\,\text{cosec}^2\theta\ \dfrac{d\theta}{dt}$

$\Rightarrow -v = -h\,\text{cosec}^2\theta \dfrac{d\theta}{dt}$ [as x is decreasing $\dfrac{dx}{dt} = -v$]

or $\dfrac{u\sec\theta}{h\text{cosec}^2\theta} = \dfrac{d\theta}{dt}$ ....(using (1)) ....(6)

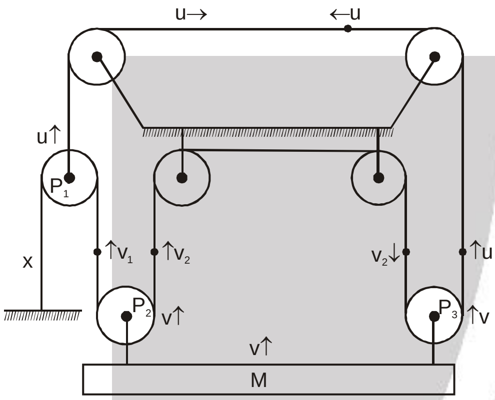
using (2) , (5) and (6) we get

$u\sec\theta\tan\theta\left(\dfrac{u\sec\theta}{h\text{cosec}^2\theta}\right)\tan\theta = g$

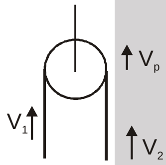
putting values of u, h & g we get.

$\tan^4\theta = 1 \Rightarrow \theta = \dfrac{\pi}{4}$ **Ans.** $\theta = \dfrac{\pi}{4}$

16.



$V_p = \dfrac{V_1 + V_2}{2}$



Pulley P_1

$u = \dfrac{0 + v_1}{2}$ .... (1)

Pulley P_2

$v = \dfrac{v_1 + v_2}{2} \Rightarrow 2V = V_1 + V_2$ ... (2)

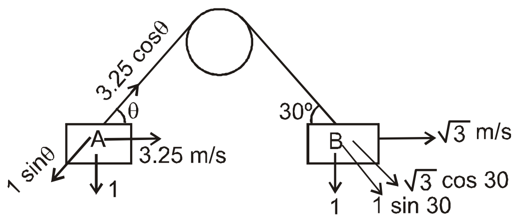
Pulley P_3 $v = \dfrac{-v_2 + u}{2}$ .... (3)

Eliminate V_1 & V_2 to get

$\Rightarrow 2u + u - 2u = 2v \Rightarrow 3u = 4v$

$v = \dfrac{3}{4}u$ **Ans.**

17. Solving problem in the frame of pulley



$3.25\cos\theta - 1\sin\theta = \sqrt{3}\cos 30 + 1\sin 30$

$3.25 \cos\theta - \sin\theta = \frac{3}{2} + \frac{1}{2}$

$3.25 \cos\theta - \sin\theta = 2$

$13 \cos \theta - 4 \sin \theta = 8$

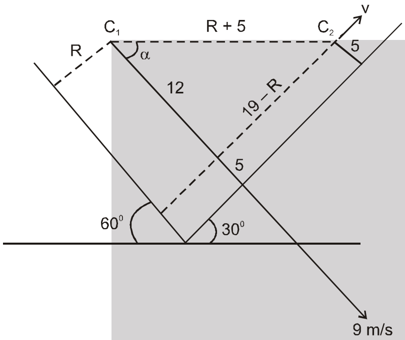
$13\sqrt{1-\sin^2\theta} = 8 + 4 \sin\theta$

$169 - 169 \sin^2\theta = 64 + 16 \sin^2\theta + 64 \sin\theta$

$185 \sin^2\theta + 64 \sin\theta - 105 = 0$

$\Rightarrow \sin \theta = \frac{3}{5} \qquad \Rightarrow \tan \theta = \frac{3}{4}$.

18.



$9 \cos\alpha = v \sin\alpha \quad \rightarrow \quad$ (i)

$\frac{19-R}{12} = \tan\alpha \quad \rightarrow \quad$ (ii)

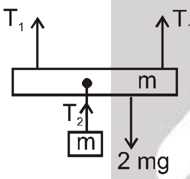
$(R + 5)^2 = (12)^2 + (19 - R)^2$ [Pythagorean]

$\Rightarrow R = 10$

Hence from (i) and (ii)

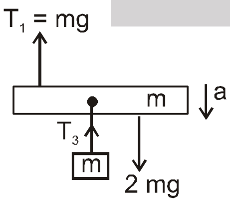
$v = 12$ m/s

19. Before cutting the spring



$T_2 = mg$

After cutting the spring

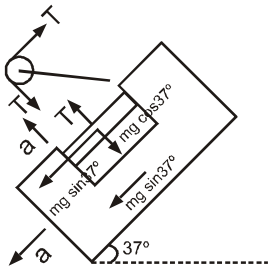


$2mg - mg = 2\,ma$

$a = g/2$

$T_3 = mg/2$

$T_2 - T_3 = mg - \frac{mg}{2} = \frac{mg}{2}$

20. 

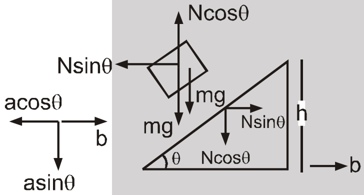
T – mg cos37º = ma

2mg sin37º – T = 2ma

$\Rightarrow$ $a = \frac{4}{3}$ m/s

$\Rightarrow$ $a_B = \frac{4}{3}$ m/s

$\Rightarrow$ $a_A = \frac{4\sqrt{2}}{3}$ m/s

21. 

N sinθ = mb

N sinθ = m(a cosθ – b)

2mg – N cosθ = ma sinθ

$\Rightarrow$ $a = \frac{4g\sin\theta}{1+\sin^2\theta}$

$\Rightarrow$ $h = \frac{1}{2} a \sin\theta\, t^2$ $\Rightarrow$ $t = \sqrt{\frac{h(1+\sin^2\theta)}{2g\sin^2\theta}}$.